14. $x = 5\tan\theta$, $dx = 5\sec^2\theta\, d\theta$, $\dfrac{1}{25}\displaystyle\int \frac{\sec\theta}{\tan^2\theta}d\theta = \frac{1}{25}\int \csc\theta\cot\theta\, d\theta = -\frac{1}{25}\csc\theta + C = -\frac{\sqrt{x^2+25}}{25x} + C$

15. $x = 3\sec\theta$, $dx = 3\sec\theta\tan\theta\, d\theta$, $\displaystyle\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C = \ln\left|\frac{1}{3}x + \frac{1}{3}\sqrt{x^2-9}\right| + C$

16. $1 + 2x^2 + x^4 = (1+x^2)^2$, $x = \tan\theta$, $dx = \sec^2\theta\, d\theta$,

$$\int \frac{1}{\sec^2\theta}d\theta = \int \cos^2\theta\, d\theta = \frac{1}{2}\int (1+\cos 2\theta)d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$$

$$= \frac{1}{2}\theta + \frac{1}{2}\sin\theta\cos\theta + C = \frac{1}{2}\tan^{-1}x + \frac{x}{2(1+x^2)} + C$$

17. $x = \dfrac{3}{2}\sec\theta$, $dx = \dfrac{3}{2}\sec\theta\tan\theta\, d\theta$,

$$\frac{3}{2}\int \frac{\sec\theta\tan\theta\, d\theta}{27\tan^3\theta} = \frac{1}{18}\int \frac{\cos\theta}{\sin^2\theta}d\theta = -\frac{1}{18}\frac{1}{\sin\theta} + C = -\frac{1}{18}\csc\theta + C = -\frac{x}{9\sqrt{4x^2-9}} + C$$

18. $x = 5\sec\theta$, $dx = 5\sec\theta\tan\theta\, d\theta$,

$$= 375\int \sec^4\theta\, d\theta$$

$$= 125\sec^2\theta\tan\theta + 250\int \sec^2\theta\, d\theta$$

$$= 125\sec^2\theta\tan\theta + 250\tan\theta + C$$

$$= x^2\sqrt{x^2-25} + 50\sqrt{x^2-25} + C$$

19. $e^x = \sin\theta$, $e^x dx = \cos\theta\, d\theta$,

$$\int \cos^2\theta\, d\theta = \frac{1}{2}\int (1+\cos 2\theta)d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C = \frac{1}{2}\sin^{-1}(e^x) + \frac{1}{2}e^x\sqrt{1-e^{2x}} + C$$

20. $u = \sin\theta$, $\displaystyle\int \frac{1}{\sqrt{2-u^2}}du = \sin^{-1}\left(\frac{\sin\theta}{\sqrt{2}}\right) + C$

27. $u = x^2 + 4$, $du = 2x\, dx$,

$\dfrac{1}{2}\displaystyle\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln(x^2+4) + C$; or $x = 2\tan\theta$, $dx = 2\sec^2\theta\, d\theta$,

$$\int \tan\theta\, d\theta = \ln|\sec\theta| + C_1 = \ln\frac{\sqrt{x^2+4}}{2} + C_1 = \ln(x^2+4)^{1/2} - \ln 2 + C_1$$

$$= \frac{1}{2}\ln(x^2+4) + C \text{ with } C = C_1 - \ln 2$$