

$$7. \frac{Ax + B}{x^2 + 5} + \frac{Cx + D}{(x^2 + 5)^2}$$

$$8. \frac{A}{x-2} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2}$$

$$9. \frac{1}{(x-4)(x+1)} = \frac{A}{x-4} + \frac{B}{x+1}; A = \frac{1}{5}, B = -\frac{1}{5} \text{ so}$$

$$\frac{1}{5} \int \frac{1}{x-4} dx - \frac{1}{5} \int \frac{1}{x+1} dx = \frac{1}{5} \ln|x-4| - \frac{1}{5} \ln|x+1| + C = \frac{1}{5} \ln \left| \frac{x-4}{x+1} \right| + C$$

$$10. \frac{1}{(x+1)(x-7)} = \frac{A}{x+1} + \frac{B}{x-7}; A = -\frac{1}{8}, B = \frac{1}{8} \text{ so}$$

$$-\frac{1}{8} \int \frac{1}{x+1} dx + \frac{1}{8} \int \frac{1}{x-7} dx = -\frac{1}{8} \ln|x+1| + \frac{1}{8} \ln|x-7| + C = \frac{1}{8} \ln \left| \frac{x-7}{x+1} \right| + C$$

$$11. \frac{11x + 17}{(2x-1)(x+4)} = \frac{A}{2x-1} + \frac{B}{x+4}; A = 5, B = 3 \text{ so}$$

$$5 \int \frac{1}{2x-1} dx + 3 \int \frac{1}{x+4} dx = \frac{5}{2} \ln|2x-1| + 3 \ln|x+4| + C$$

$$12. \frac{5x-5}{(x-3)(3x+1)} = \frac{A}{x-3} + \frac{B}{3x+1}; A = 1, B = 2 \text{ so}$$

$$\int \frac{1}{x-3} dx + 2 \int \frac{1}{3x+1} dx = \ln|x-3| + \frac{2}{3} \ln|3x+1| + C$$

$$13. \frac{2x^2 - 9x - 9}{x(x+3)(x-3)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-3}; A = 1, B = 2, C = -1 \text{ so}$$

$$\int \frac{1}{x} dx + 2 \int \frac{1}{x+3} dx - \int \frac{1}{x-3} dx = \ln|x| + 2 \ln|x+3| - \ln|x-3| + C = \ln \left| \frac{x(x+3)^2}{x-3} \right| + C$$

Note that the symbol C has been recycled; to save space this recycling is usually not mentioned.

$$14. \frac{1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}; A = -1, B = \frac{1}{2}, C = \frac{1}{2} \text{ so}$$

$$\begin{aligned} -\int \frac{1}{x} dx + \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{x-1} dx &= -\ln|x| + \frac{1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1| + C \\ &= \frac{1}{2} \ln \left| \frac{(x+1)(x-1)}{x^2} \right| + C = \frac{1}{2} \ln \left| \frac{x^2-1}{x^2} \right| + C \end{aligned}$$

$$15. \frac{x^2-8}{x+3} = x-3 + \frac{1}{x+3}, \int \left(x-3 + \frac{1}{x+3} \right) dx = \frac{1}{2}x^2 - 3x + \ln|x+3| + C$$

$$16. \frac{x^2+1}{x-1} = x+1 + \frac{2}{x-1}, \int \left(x+1 + \frac{2}{x-1} \right) dx = \frac{1}{2}x^2 + x + 2 \ln|x-1| + C$$