

$$19. \frac{x^5 + x^2 + 2}{x^3 - x} = x^2 + 1 + \frac{x^2 + x + 2}{x^3 - x},$$

$$\frac{x^2 + x + 2}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1}; A = -2, B = 1, C = 2 \text{ so}$$

$$\begin{aligned} \int (x^2 + 1)dx - \int \frac{2}{x}dx + \int \frac{1}{x+1}dx + \int \frac{2}{x-1}dx \\ = \frac{1}{3}x^3 + x - 2\ln|x| + \ln|x+1| + 2\ln|x-1| + C = \frac{1}{3}x^3 + x + \ln \left| \frac{(x+1)(x-1)^2}{x^2} \right| + C \end{aligned}$$

$$20. \frac{x^5 - 4x^3 + 1}{x^3 - 4x} = x^2 + \frac{1}{x^3 - 4x},$$

$$\frac{1}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}; A = -\frac{1}{4}, B = +\frac{1}{8}, C = +\frac{1}{8} \text{ so}$$

$$\begin{aligned} \int x^2 dx - \frac{1}{4} \int \frac{1}{x} dx + \frac{1}{8} \int \frac{1}{x+2} dx + \frac{1}{8} \int \frac{1}{x-2} dx \\ = \frac{1}{3}x^3 - \frac{1}{4} \ln|x| + \frac{1}{8} \ln|x+2| + \frac{1}{8} \ln|x-2| + C \end{aligned}$$

$$21. \frac{2x^2 + 3}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}; A = 3, B = -1, C = 5 \text{ so}$$

$$3 \int \frac{1}{x} dx - \int \frac{1}{x-1} dx + 5 \int \frac{1}{(x-1)^2} dx = 3\ln|x| - \ln|x-1| - 5/(x-1) + C$$

$$22. \frac{3x^2 - x + 1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}; A = 0, B = -1, C = 3 \text{ so}$$

$$- \int \frac{1}{x^2} dx + 3 \int \frac{1}{x-1} dx = 1/x + 3\ln|x-1| + C$$

$$23. \frac{2x^2 - 10x + 4}{(x+1)(x-3)^2} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}; A = 1, B = 1, C = -2 \text{ so}$$

$$\int \frac{1}{x+1} dx + \int \frac{1}{x-3} dx - \int \frac{2}{(x-3)^2} dx = \ln|x+1| + \ln|x-3| + \frac{2}{x-3} + C_1$$