27. $\dfrac{2x^2-1}{(4x-1)(x^2+1)} = \dfrac{A}{4x-1} + \dfrac{Bx+C}{x^2+1}$; $A = -14/17$, $B = 12/17$, $C = 3/17$ so

$$\int \frac{2x^2-1}{(4x-1)(x^2+1)}dx = -\frac{7}{34}\ln|4x-1| + \frac{6}{17}\ln(x^2+1) + \frac{3}{17}\tan^{-1}x + C.$$

28. $\dfrac{1}{x(x^2+2)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+2}; A = \dfrac{1}{2}, B = -\dfrac{1}{2}, C = 0$ so

$$\int \frac{1}{x^3+2x}\,dx = \frac{1}{2}\ln|x| - \frac{1}{4}\ln(x^2+2) + C = \frac{1}{4}\ln\frac{x^2}{x^2+2} + C$$

29. $\dfrac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{x^2+3}$; $A = 0$, $B = 3$, $C = 1$, $D = 0$ so

$$\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}\,dx = 3\tan^{-1}x + \frac{1}{2}\ln(x^2+3) + C$$

30. $\dfrac{x^3+x^2+x+2}{(x^2+1)(x^2+2)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{x^2+2}$; $A = D = 0$, $B = C = 1$ so

$$\int \frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)}\,dx = \tan^{-1}x + \frac{1}{2}\ln(x^2+2) + C$$

31. $\dfrac{x^3-2x^2+2x-2}{x^2+1} = x - 2 + \dfrac{x}{x^2+1}$,

$$\int \frac{x^3-3x^2+2x-3}{x^2+1}\,dx = \frac{1}{2}x^2 - 2x + \frac{1}{2}\ln(x^2+1) + C$$

32. $\dfrac{x^4+6x^3+10x^2+x}{x^2+6x+10} = x^2 + \dfrac{x}{x^2+6x+10}$,

$$\int \frac{x}{x^2+6x+10}\,dx = \int \frac{x}{(x+3)^2+1}\,dx = \int \frac{u-3}{u^2+1}du, \qquad u = x+3$$

$$= \frac{1}{2}\ln(u^2+1) - 3\tan^{-1}u + C_1$$

so $\displaystyle\int \frac{x^4+6x^3+10x^2+x}{x^2+6x+10}\,dx = \frac{1}{3}x^3 + \frac{1}{2}\ln(x^2+6x+10) - 3\tan^{-1}(x+3) + C$

33. Let $x = \sin\theta$ to get $\displaystyle\int \frac{1}{x^2+4x-5}\,dx$, and $\dfrac{1}{(x+5)(x-1)} = \dfrac{A}{x+5} + \dfrac{B}{x-1}$; $A = -1/6$, $B = 1/6$ so we get $\displaystyle -\frac{1}{6}\int \frac{1}{x+5}\,dx + \frac{1}{6}\int \frac{1}{x-1}\,dx = \frac{1}{6}\ln\left|\frac{x-1}{x+5}\right| + C = \frac{1}{6}\ln\left(\frac{1-\sin\theta}{5+\sin\theta}\right) + C.$

34. Let $x = e^t$; then $\displaystyle\int \frac{e^t}{e^{2t}-4}dt = \int \frac{1}{x^2-4}\,dx,$

$\dfrac{1}{(x+2)(x-2)} = \dfrac{A}{x+2} + \dfrac{B}{x-2}$; $A = -1/4$, $B = 1/4$ so

$$-\frac{1}{4}\int \frac{1}{x+2}\,dx + \frac{1}{4}\int \frac{1}{x-2}\,dx = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C = \frac{1}{4}\ln\left|\frac{e^t-2}{e^t+2}\right| + C.$$