7–8 The first-order linear equations in these exercises can be rewritten as first-order separable equations. Solve the equations using both the method of integrating factors and the method of separation of variables, and determine whether the solutions produced are the same.

7. (a) $\dfrac{dy}{dx} + 3y = 0$ (b) $\dfrac{dy}{dt} - 2y = 0$

8. (a) $\dfrac{dy}{dx} - 4xy = 0$ (b) $\dfrac{dy}{dt} + y = 0$

9–14 Solve the differential equation by the method of integrating factors.

9. $\dfrac{dy}{dx} + 4y = e^{-3x}$

10. $\dfrac{dy}{dx} + 2xy = x$

11. $y' + y = \cos(e^x)$

12. $2\dfrac{dy}{dx} + 4y = 1$

13. $(x^2+1)\dfrac{dy}{dx} + xy = 0$

14. $\dfrac{dy}{dx} + y + \dfrac{1}{1-e^x} = 0$

15–24 Solve the differential equation by separation of variables. Where reasonable, express the family of solutions as explicit functions of x.

15. $\dfrac{dy}{dx} = \dfrac{y}{x}$

16. $\dfrac{dy}{dx} = 2(1+y^2)x$

17. $\dfrac{\sqrt{1+x^2}}{1+y}\dfrac{dy}{dx} = -x$

18. $(1+x^4)\dfrac{dy}{dx} = \dfrac{x^3}{y}$

19. $(2+2y^2)y' = e^x y$

20. $y' = -xy$

21. $e^{-y}\sin x - y'\cos^2 x = 0$

22. $y' - (1+x)(1+y^2) = 0$

23. $\dfrac{dy}{dx} - \dfrac{y^2-y}{\sin x} = 0$

24. $y - \dfrac{dy}{dx}\sec x = 0$

27–32 Solve the initial-value problem by any method.

27. $\dfrac{dy}{dx} - 2xy = 2x, \quad y(0) = 3$

28. $\dfrac{dy}{dt} + y = 2, \quad y(0) = 1$

29. $y' = \dfrac{3x^2}{2y+\cos y}, \quad y(0) = \pi$

30. $y' - xe^y = 2e^y, \quad y(0) = 0$

31. $\dfrac{dy}{dt} = \dfrac{2t+1}{2y-2}, \quad y(0) = -1$

43. At time $t = 0$, a tank contains 25 oz of salt dissolved in 50 gal of water. Then brine containing 4 oz of salt per gallon of brine is allowed to enter the tank at a rate of 2 gal/min and the mixed solution is drained from the tank at the same rate.
(a) How much salt is in the tank at an arbitrary time t?
(b) How much salt is in the tank after 25 min?

44. A tank initially contains 200 gal of pure water. Then at time $t = 0$ brine containing 5 lb of salt per gallon of brine is allowed to enter the tank at a rate of 20 gal/min and the mixed solution is drained from the tank at the same rate.
(a) How much salt is in the tank at an arbitrary time t?
(b) How much salt is in the tank after 30 min?

45. A tank with a 1000-gal capacity initially contains 500 gal of water that is polluted with 50 lb of particulate matter. At time $t = 0$, pure water is added at a rate of 20 gal/min and the mixed solution is drained off at a rate of 10 gal/min. How much particulate matter is in the tank when it reaches the point of overflowing?

46. The water in a polluted lake initially contains 1 lb of mercury salts per 100,000 gal of water. The lake is circular with diameter 30 m and uniform depth 3 m. Polluted water is pumped from the lake at a rate of 1000 gal/h and is replaced with fresh water at the same rate. Construct a table that shows the amount of mercury in the lake (in lb) at the end of each hour over a 12-hour period. Discuss any assumptions you made. [Use 264 gal/m^3.]

47. (a) Use the method of integrating factors to derive solution (27) to the initial-value problem (25). [*Note:* Keep in mind that c, m, and g are constants.]
(b) Show that (27) can be expressed in terms of the terminal speed (29) as
$$v(t) = e^{-gt/v_\tau}(v_0 + v_\tau) - v_\tau$$
(c) Show that if $s(0) = s_0$, then the position function of the object can be expressed as
$$s(t) = s_0 - v_\tau t + \frac{v_\tau}{g}(v_0 + v_\tau)(1 - e^{-gt/v_\tau})$$

48. Suppose a fully equipped sky diver weighing 240 lb has a terminal speed of 120 ft/s with a closed parachute and 24 ft/s with an open parachute. Suppose further that this sky diver is dropped from an airplane at an altitude of 10,000 ft, falls for 25 s with a closed parachute, and then falls the rest of the way with an open parachute.
(a) Assuming that the sky diver's initial vertical velocity is zero, use Exercise 47 to find the sky diver's vertical velocity and height at the time the parachute opens. [Take $g = 32$ ft/s^2.]
(b) Use a calculating utility to find a numerical solution for the total time that the sky diver is in the air.

49. The accompanying figure is a schematic diagram of a basic RL series electrical circuit that contains a power source with a time-dependent voltage of $V(t)$ volts (V), a resistor with a constant resistance of R ohms (Ω), and an inductor with a constant inductance of L henrys (H). If you don't know anything about electrical circuits, don't worry; all you need to know is that electrical theory states that a current of $I(t)$ amperes (A) flows through the circuit where $I(t)$ satisfies the differential equation
$$L\frac{dI}{dt} + RI = V(t)$$
(a) Find $I(t)$ if $R = 10\,\Omega$, $L = 5$ H, V is a constant 20 V, and $I(0) = 0$ A.
(b) What happens to the current over a long period of time?

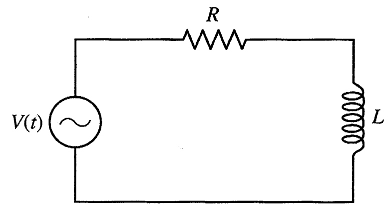

Figure Ex-49

50. Find $I(t)$ for the electrical circuit in Exercise 49 if $R = 6\,\Omega$, $L = 3$ H, $V(t) = 3\sin t$ V, and $I(0) = 15$ A.

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