

25. $\frac{dy}{dx} + \frac{1}{x}y = 1$, $\mu = e^{\int(1/x)dx} = e^{\ln x} = x$, $\frac{d}{dx}[xy] = x$, $xy = \frac{1}{2}x^2 + C$, $y = x/2 + C/x$

(a) $2 = y(1) = \frac{1}{2} + C$, $C = \frac{3}{2}$, $y = x/2 + 3/(2x)$

(b) $2 = y(-1) = -1/2 - C$, $C = -5/2$, $y = x/2 - 5/(2x)$

26. $\frac{dy}{y} = x dx$, $\ln|y| = \frac{x^2}{2} + C_1$, $y = \pm e^{C_1} e^{x^2/2} = C e^{x^2/2}$

(a) $1 = y(0) = C$ so $C = 1$, $y = e^{x^2/2}$

(b) $\frac{1}{2} = y(0) = C$, so $y = \frac{1}{2}e^{x^2/2}$

27. $\mu = e^{-2\int x dx} = e^{-x^2}$, $e^{-x^2} y = \int 2xe^{-x^2} dx = -e^{-x^2} + C$,
 $y = -1 + C e^{x^2}$, $3 = -1 + C$, $C = 4$, $y = -1 + 4e^{x^2}$

28. $\mu = e^{\int dt} = e^t$, $e^t y = \int 2e^t dt = 2e^t + C$, $y = 2 + C e^{-t}$, $1 = 2 + C$, $C = -1$, $y = 2 - e^{-t}$

29. $(2y + \cos y) dy = 3x^2 dx$, $y^2 + \sin y = x^3 + C$, $\pi^2 + \sin \pi = C$, $C = \pi^2$,
 $y^2 + \sin y = x^3 + \pi^2$

30. $\frac{dy}{dx} = (x+2)e^y$, $e^{-y} dy = (x+2)dx$, $-e^{-y} = \frac{1}{2}x^2 + 2x + C$, $-1 = C$,
 $-e^{-y} = \frac{1}{2}x^2 + 2x - 1$, $e^{-y} = -\frac{1}{2}x^2 - 2x + 1$, $y = -\ln\left(1 - 2x - \frac{1}{2}x^2\right)$

31. $2(y-1) dy = (2t+1) dt$, $y^2 - 2y = t^2 + t + C$, $1 + 2 = C$, $C = 3$, $y^2 - 2y = t^2 + t + 3$