

43. $\frac{dy}{dt} = \text{rate in} - \text{rate out}$, where y is the amount of salt at time t ,

$$\frac{dy}{dt} = (4)(2) - \left(\frac{y}{50}\right)(2) = 8 - \frac{1}{25}y, \text{ so } \frac{dy}{dt} + \frac{1}{25}y = 8 \text{ and } y(0) = 25.$$

$$\mu = e^{\int(1/25)dt} = e^{t/25}, e^{t/25}y = \int 8e^{t/25}dt = 200e^{t/25} + C,$$

$$y = 200 + Ce^{-t/25}, 25 = 200 + C, C = -175,$$

(a) $y = 200 - 175e^{-t/25}$ oz

(b) when $t = 25$, $y = 200 - 175e^{-1} \approx 136$ oz

44. $\frac{dy}{dt} = (5)(20) - \frac{y}{200}(20) = 100 - \frac{1}{10}y$, so $\frac{dy}{dt} + \frac{1}{10}y = 100$ and $y(0) = 0$.

$$\mu = e^{\int \frac{1}{10}dt} = e^{t/10}, e^{t/10}y = \int 100e^{t/10}dt = 1000e^{t/10} + C,$$

$$y = 1000 + Ce^{-t/10}, 0 = 1000 + C, C = -1000;$$

(a) $y = 1000 - 1000e^{-t/10}$ lb

(b) when $t = 30$, $y = 1000 - 1000e^{-3} \approx 950$ lb

45. The volume V of the (polluted) water is $V(t) = 500 + (20 - 10)t = 500 + 10t$;
if $y(t)$ is the number of pounds of particulate matter in the water,

then $y(0) = 50$, and $\frac{dy}{dt} = 0 - 10\frac{y}{V} = -\frac{10y}{50+t}$, $\frac{dy}{dt} + \frac{10y}{50+t} = 0$; $\mu = e^{\int \frac{10}{50+t}dt} = 50 + t$;

$$\frac{d}{dt}[(50+t)y] = 0, (50+t)y = C, 2500 = 50y(0) = C, y(t) = 2500/(50+t).$$

The tank reaches the point of overflowing when $V = 500 + 10t = 1000$, $t = 50$ min, so
 $y = 2500/(50 + 50) = 25$ lb.

46. The volume of the lake (in gallons) is $V = 264\pi r^2 h = 264\pi(15)^2 3 = 178,200\pi$ gals. Let $y(t)$ denote the number of pounds of mercury salts at time t , then $\frac{dy}{dt} = 0 - 10^3 \frac{y}{V} = -\frac{y}{178.2\pi}$ lb/h and $y_0 = 10^{-5}V = 1.782\pi$ lb; $\frac{dy}{y} = -\frac{dt}{178.2\pi}$, $\ln y = -\frac{t}{178.2\pi} + C_1$, $y = Ce^{-t/(178.2\pi)}$, and $C = y(0) = y_0 10^{-5} V = 1.782\pi$, $y = 1.782\pi e^{-t/(178.2\pi)}$ lb of mercury salts.

t	1	2	3	4	5	6	7	8	9	10	11	12
$y(t)$	5.588	5.578	5.568	5.558	5.549	5.539	5.529	5.519	5.509	5.499	5.489	5.480