

$$20. y = (\ln x)^{\cos x} \Rightarrow \ln y = \cos x \ln(\ln x) \Rightarrow \frac{y'}{y} = \cos x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + (\ln \ln x)(-\sin x) \Rightarrow$$

$$y' = (\ln x)^{\cos x} \left( \frac{\cos x}{x \ln x} - \sin x \ln \ln x \right)$$

$$21. y = \ln \sin x - \frac{1}{2} \sin^2 x \Rightarrow y' = \frac{1}{\sin x} \cdot \cos x - \frac{1}{2} \cdot 2 \sin x \cdot \cos x = \cot x - \sin x \cos x$$

$$22. y = \frac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5} \Rightarrow$$

$$\ln y = \ln \frac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5} = \ln(x^2 + 1)^4 - \ln[(2x + 1)^3(3x - 1)^5]$$

$$= 4 \ln(x^2 + 1) - [\ln(2x + 1)^3 + \ln(3x - 1)^5] = 4 \ln(x^2 + 1) - 3 \ln(2x + 1) - 5 \ln(3x - 1) \Rightarrow$$

$$\frac{y'}{y} = 4 \cdot \frac{1}{x^2 + 1} \cdot 2x - 3 \cdot \frac{1}{2x + 1} \cdot 2 - 5 \cdot \frac{1}{3x - 1} \cdot 3 \Rightarrow$$

$$y' = \frac{(x^2 + 1)^4}{(2x + 1)^3(3x - 1)^5} \left( \frac{8x}{x^2 + 1} - \frac{6}{2x + 1} - \frac{15}{3x - 1} \right). \text{ [The answer could be simplified to}$$

$$y' = -\frac{(x^2 + 56x + 9)(x^2 + 1)^3}{(2x + 1)^4(3x - 1)^6}, \text{ but this is unnecessary.]}$$

$$23. y = x \tan^{-1}(4x) \Rightarrow y' = x \cdot \frac{1}{1 + (4x)^2} \cdot 4 + \tan^{-1}(4x) \cdot 1 = \frac{4x}{1 + 16x^2} + \tan^{-1}(4x)$$

$$24. y = e^{\cos x} + \cos(e^x) \Rightarrow y' = e^{\cos x}(-\sin x) + [-\sin(e^x) \cdot e^x] = -\sin x e^{\cos x} - e^x \sin(e^x)$$

$$25. y = \ln |\sec 5x + \tan 5x| \Rightarrow$$

$$y' = \frac{1}{\sec 5x + \tan 5x} (\sec 5x \tan 5x \cdot 5 + \sec^2 5x \cdot 5) = \frac{5 \sec 5x (\tan 5x + \sec 5x)}{\sec 5x + \tan 5x} = 5 \sec 5x$$

$$26. y = 10^{\tan \pi \theta} \Rightarrow y' = 10^{\tan \pi \theta} \cdot \ln 10 \cdot \sec^2 \pi \theta \cdot \pi = \pi (\ln 10) 10^{\tan \pi \theta} \sec^2 \pi \theta$$

$$27. y = \cot(3x^2 + 5) \Rightarrow y' = -\csc^2(3x^2 + 5)(6x) = -6x \csc^2(3x^2 + 5)$$

$$28. y = \ln \left| \frac{x^2 - 4}{2x + 5} \right| = \ln |x^2 - 4| - \ln |2x + 5| \Rightarrow y' = \frac{2x}{x^2 - 4} - \frac{2}{2x + 5} \text{ or } \frac{2(x + 1)(x + 4)}{(x + 2)(x - 2)(2x + 5)}$$

$$29. y = \sin(\tan \sqrt{1 + x^3}) \Rightarrow y' = \cos(\tan \sqrt{1 + x^3}) (\sec^2 \sqrt{1 + x^3}) [3x^2 / (2\sqrt{1 + x^3})]$$

$$30. y = \arctan(\arcsin \sqrt{x}) \Rightarrow y' = \frac{1}{1 + (\arcsin \sqrt{x})^2} \cdot \frac{1}{\sqrt{1 - x}} \cdot \frac{1}{2\sqrt{x}}$$