51. $y = [\ln(x+4)]^2 \Rightarrow y' = 2[\ln(x+4)]^1 \cdot \dfrac{1}{x+4} \cdot 1 = 2\dfrac{\ln(x+4)}{x+4}$ and $y' = 0 \Leftrightarrow \ln(x+4) = 0 \Leftrightarrow$ $x + 4 = e^0 \Rightarrow x + 4 = 1 \Leftrightarrow x = -3$, so the tangent is horizontal at the point $(-3, 0)$.

52. (a) The line $x - 4y = 1$ has slope $\frac{1}{4}$. A tangent to $y = e^x$ has slope $\frac{1}{4}$ when $y' = e^x = \frac{1}{4} \Rightarrow$ $x = \ln\frac{1}{4} = -\ln 4$. Since $y = e^x$, the y-coordinate is $\frac{1}{4}$ and the point of tangency is $\left(-\ln 4, \frac{1}{4}\right)$. Thus, an equation of the tangent line is $y - \frac{1}{4} = \frac{1}{4}(x + \ln 4)$ or $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$.

(b) The slope of the tangent at the point (a, e^a) is $\left.\dfrac{d}{dx} e^x \right|_{x=a} = e^a$. Thus, an equation of the tangent line is $y - e^a = e^a(x - a)$. We substitute $x = 0$, $y = 0$ into this equation, since we want the line to pass through the origin: $0 - e^a = e^a(0 - a) \Leftrightarrow -e^a = e^a(-a) \Leftrightarrow a = 1$. So an equation of the tangent line at the point $(a, e^a) = (1, e)$ is $y - e = e(x - 1)$ or $y = ex$.

53. $x^2 + 2y^2 = 1 \Rightarrow 2x + 4yy' = 0 \Rightarrow y' = -x/(2y) = 1 \Leftrightarrow x = -2y$. Since the points lie on the ellipse, we have $(-2y)^2 + 2y^2 = 1 \Rightarrow 6y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{6}}$. The points are $\left(-\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $\left(\frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$.

54. (a) $f(x) = \dfrac{\ln x}{x} \Rightarrow f'(x) = \dfrac{x(1/x) - (\ln x) \cdot 1}{x^2} = \dfrac{1 - \ln x}{x^2}$.
$f'(x) > 0 \Rightarrow 1 - \ln x > 0 \Rightarrow \ln x < 1 \Rightarrow x < e$. Since the domain of f is $(0, \infty)$, f is increasing on $(0, e)$.

(b) $f''(x) = \dfrac{x^2(-1/x) - (1 - \ln x)(2x)}{(x^2)^2} = \dfrac{x[-1 - 2(1 - \ln x)]}{x^4} = \dfrac{2\ln x - 3}{x^3}$.
$f''(x) > 0 \Rightarrow 2\ln x - 3 > 0 \Rightarrow \ln x > \frac{3}{2} \Rightarrow x > e^{3/2} \approx 4.48$.
f is concave upward on $\left(e^{3/2}, \infty\right)$.

55. $s(t) = Ae^{-ct}\cos(\omega t + \delta) \Rightarrow$

$$\begin{aligned} v(t) = s'(t) &= A\{e^{-ct}[-\omega\sin(\omega t + \delta)] + \cos(\omega t + \delta)(-ce^{-ct})\} \\ &= -Ae^{-ct}[\omega\sin(\omega t + \delta) + c\cos(\omega t + \delta)] \Rightarrow \\ a(t) = v'(t) &= -A\{e^{-ct}[\omega^2\cos(\omega t + \delta) - c\omega\sin(\omega t + \delta)] \\ &\qquad + [\omega\sin(\omega t + \delta) + c\cos(\omega t + \delta)](-ce^{-ct})\} \\ &= -Ae^{-ct}[\omega^2\cos(\omega t + \delta) - c\omega\sin(\omega t + \delta) - c\omega\sin(\omega t + \delta) - c^2\cos(\omega t + \delta)] \\ &= -Ae^{-ct}[(\omega^2 - c^2)\cos(\omega t + \delta) - 2c\omega\sin(\omega t + \delta)] \\ &= Ae^{-ct}[(c^2 - \omega^2)\cos(\omega t + \delta) + 2c\omega\sin(\omega t + \delta)] \end{aligned}$$

56. (a) $y = t^3 - 12t + 3 \Rightarrow v(t) = y' = 3t^2 - 12 \Rightarrow a(t) = v'(t) = 6t$

(b) $v(t) = 3(t^2 - 4) > 0$ when $t > 2$, so it moves upward when $t > 2$ and downward when $0 \le t < 2$.

(c) Distance upward $= y(3) - y(2) = -6 - (-13) = 7$,
Distance downward $= y(0) - y(2) = 3 - (-13) = 16$. Total distance $= 7 + 16 = 23$.