STEWART 4.5

5–36 ■ Find the limit. Use l'Hospital's Rule where appropriate. If there is a more elementary method, consider using it. If l'Hospital's Rule doesn't apply, explain why.

5. $\lim_{x \to -1} \frac{x^2 - 1}{x + 1}$

6. $\lim_{x \to 1} \frac{x^a - 1}{x^b - 1}$

7. $\lim_{x \to 0} \frac{e^x - 1}{\sin x}$

8. $\lim_{x \to 0} \frac{x + \tan x}{\sin x}$

9. $\lim_{x \to 0} \frac{\tan px}{\tan qx}$

10. $\lim_{x \to \pi} \frac{\tan x}{x}$

11. $\lim_{x \to 0^+} \frac{\ln x}{x}$

12. $\lim_{x \to \infty} \frac{\ln \ln x}{x}$

13. $\lim_{t \to 0} \frac{5^t - 3^t}{t}$

14. $\lim_{x \to \infty} \frac{e^x}{x^3}$

15. $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$

16. $\lim_{x \to 0} \frac{\cos mx - \cos nx}{x^2}$

17. $\lim_{x \to 0} \frac{\sin^{-1} x}{x}$

18. $\lim_{x \to 0} \frac{x}{\tan^{-1}(4x)}$

19. $\lim_{x \to \infty} \frac{x}{\ln(1 + 2e^x)}$

20. $\lim_{x \to 0} \frac{1 - e^{-2x}}{\sec x}$

21. $\lim_{x \to 0^+} \sqrt{x} \ln x$

22. $\lim_{x \to -\infty} x^2 e^x$

23. $\lim_{x \to \infty} e^{-x} \ln x$

24. $\lim_{x \to (\pi/2)^-} \sec 7x \cos 3x$

25. $\lim_{x \to \infty} x^3 e^{-x^2}$

26. $\lim_{x \to 1^+} (x - 1) \tan(\pi x/2)$

27. $\lim_{x \to 0} \left(\frac{1}{x} - \csc x \right)$

28. $\lim_{x \to 0} (\csc x - \cot x)$

29. $\lim_{x \to \infty} (x e^{1/x} - x)$

30. $\lim_{x \to 1} \left(\frac{1}{\ln x} - \frac{1}{x - 1} \right)$

31. $\lim_{x \to 0^+} x^{\sin x}$

32. $\lim_{x \to 0^+} (\sin x)^{\tan x}$

33. $\lim_{x \to 0} (1 - 2x)^{1/x}$

34. $\lim_{x \to \infty} \left(1 + \frac{a}{x} \right)^{bx}$

35. $\lim_{x \to 0^+} (-\ln x)^x$

36. $\lim_{x \to \infty} x^{(\ln 2)/(1 + \ln x)}$

51. Prove that

$$\lim_{x \to \infty} \frac{e^x}{x^n} = \infty$$

for any positive integer n. This shows that the exponential function approaches infinity faster than any power of x.

52. Prove that

$$\lim_{x \to \infty} \frac{\ln x}{x^p} = 0$$

for any number $p > 0$. This shows that the logarithmic function approaches ∞ more slowly than any power of x.

53. If an initial amount A_0 of money is invested at an interest rate i compounded n times a year, the value of the investment after t years is

$$A = A_0 \left(1 + \frac{i}{n} \right)^{nt}$$

If we let $n \to \infty$, we refer to the *continuous compounding* of interest. Use l'Hospital's Rule to show that if interest is compounded continuously, then the amount after n years is

$$A = A_0 e^{it}$$

54. If an object with mass m is dropped from rest, one model for its speed v after t seconds, taking air resistance into account, is

$$v = \frac{mg}{c} (1 - e^{-ct/m})$$

where g is the acceleration due to gravity and c is a positive constant. (In Chapter 7 we will be able to deduce this equation from the assumption that the air resistance is proportional to the speed of the object.)

(a) Calculate $\lim_{t \to \infty} v$. What is the meaning of this limit?

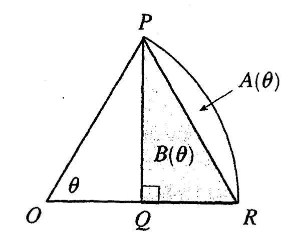
(b) For fixed t, use l'Hospital's Rule to calculate $\lim_{m \to \infty} v$. What can you conclude about the speed of a very heavy falling object?

55. The first appearance in print of l'Hospital's Rule was in the book *Analyse des Infiniment Petits* published by the Marquis de l'Hospital in 1696. This was the first calculus textbook ever published and the example that the Marquis used in that book to illustrate his rule was to find the limit of the function

$$y = \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{aax}}{a - \sqrt[4]{ax^3}}$$

as x approaches a, where $a > 0$. (At that time it was common to write aa instead of a^2.) Solve this problem.

56. The figure shows a sector of a circle with central angle θ. Let $A(\theta)$ be the area of the segment between the chord PR and the arc PR. Let $B(\theta)$ be the area of the triangle PQR. Find $\lim_{\theta \to 0^+} A(\theta)/B(\theta)$.



57. If f' is continuous, use l'Hospital's Rule to show that

$$\lim_{h \to 0} \frac{f(x + h) - f(x - h)}{2h} = f'(x)$$

Explain the meaning of this equation with the aid of a diagram.