

7. $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$ is an indeterminate form of type $\frac{0}{0}$, so we'll apply l'Hospital's Rule.

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x}{\cos x} = \frac{1}{1} = 1$$

8. $\lim_{x \rightarrow 0} \frac{x + \tan x}{\sin x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1 + \sec^2 x}{\cos x} = \frac{1 + 1^2}{1} = 2$

9. This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow 0} \frac{\tan px}{\tan qx} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{p \sec^2 px}{q \sec^2 qx} = \frac{p(1)^2}{q(1)^2} = \frac{p}{q}$

10. $\lim_{x \rightarrow \pi} \frac{\tan x}{x} = \frac{\tan \pi}{\pi} = \frac{0}{\pi} = 0$

11. $\lim_{x \rightarrow 0^+} [(\ln x)/x] = -\infty$ since $\ln x \rightarrow -\infty$ as $x \rightarrow 0^+$ and dividing by small values of x just increases the magnitude of the quotient $(\ln x)/x$. l'Hospital's Rule does not apply.

12. $\lim_{x \rightarrow \infty} \frac{\ln \ln x}{x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{1} = \lim_{x \rightarrow \infty} \frac{1}{x \ln x} = 0$

13. This limit has the form $\frac{0}{0}$. $\lim_{t \rightarrow 0} \frac{5^t - 3^t}{t} \stackrel{H}{=} \lim_{t \rightarrow 0} \frac{5^t \ln 5 - 3^t \ln 3}{1} = \ln 5 - \ln 3 = \ln \frac{5}{3}$

14. $\lim_{x \rightarrow \infty} \frac{e^x}{x^3} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{3x^2} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{6x} \stackrel{H}{=} \lim_{x \rightarrow \infty} \frac{e^x}{6} = \infty$

15. This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$

16. $\lim_{x \rightarrow 0} \frac{\cos mx - \cos nx}{x^2} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{-m \sin mx + n \sin nx}{2x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{-m^2 \cos mx + n^2 \cos nx}{2} = \frac{1}{2}(n^2 - m^2)$

17. This limit has the form $\frac{0}{0}$. $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1/\sqrt{1-x^2}}{1} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{1-x^2}} = \frac{1}{1} = 1$

18. $\lim_{x \rightarrow 0} \frac{x}{\tan^{-1}(4x)} \stackrel{H}{=} \lim_{x \rightarrow 0} \frac{1}{\frac{1}{1+(4x)^2} \cdot 4} = \lim_{x \rightarrow 0} \frac{1+16x^2}{4} = \frac{1}{4}$