19. This limit has the form $\frac{\infty}{\infty}$. $\lim_{x\to\infty}\frac{x}{\ln(1+2e^x)} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{1}{\frac{1}{1+2e^x}\cdot 2e^x} = \lim_{x\to\infty}\frac{1+2e^x}{2e^x} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{2e^x}{2e^x} = 1$

20. $\lim_{x\to 0}\frac{1-e^{-2x}}{\sec x} = \frac{1-1}{1} = 0$. L'Hospital's Rule does not apply.

21. This limit has the form $0\cdot(-\infty)$. We need to write this product as a quotient, but keep in mind that we will have to differentiate both the numerator and the denominator. If we differentiate $\frac{1}{\ln x}$, we get a complicated expression that results in a more difficult limit. Instead we write the quotient as $\frac{\ln x}{x^{-1/2}}$.

$$\lim_{x\to 0^+}\sqrt{x}\ln x = \lim_{x\to 0^+}\frac{\ln x}{x^{-1/2}} \overset{\text{H}}{=} \lim_{x\to 0^+}\frac{1/x}{-\frac{1}{2}x^{-3/2}}\cdot\frac{-2x^{3/2}}{-2x^{3/2}} = \lim_{x\to 0^+}(-2\sqrt{x}) = 0$$

22. $\lim_{x\to-\infty}x^2e^x = \lim_{x\to-\infty}\frac{x^2}{e^{-x}} \overset{\text{H}}{=} \lim_{x\to-\infty}\frac{2x}{-e^{-x}} \overset{\text{H}}{=} \lim_{x\to-\infty}\frac{2}{e^{-x}} = \lim_{x\to-\infty}2e^x = 0$

23. $\lim_{x\to\infty}e^{-x}\ln x = \lim_{x\to\infty}\frac{\ln x}{e^x} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{1/x}{e^x} = \lim_{x\to\infty}\frac{1}{xe^x} = 0$

24. $\lim_{x\to(\pi/2)^-}\sec 7x\cos 3x = \lim_{x\to(\pi/2)^-}\frac{\cos 3x}{\cos 7x} \overset{\text{H}}{=} \lim_{x\to(\pi/2)^-}\frac{-3\sin 3x}{-7\sin 7x} = \frac{3(-1)}{7(-1)} = \frac{3}{7}$

25. $\lim_{x\to\infty}x^3e^{-x^2} = \lim_{x\to\infty}\frac{x^3}{e^{x^2}} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{3x^2}{2xe^{x^2}} = \lim_{x\to\infty}\frac{3x}{2e^{x^2}} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{3}{4xe^{x^2}} = 0$

26. $\lim_{x\to 1^+}(x-1)\tan(\pi x/2) = \lim_{x\to 1^+}\frac{x-1}{\cot(\pi x/2)} \overset{\text{H}}{=} \lim_{x\to 1^+}\frac{1}{-\csc^2(\pi x/2)\frac{\pi}{2}} = -\frac{2}{\pi}$

27. $\lim_{x\to 0}\left(\frac{1}{x}-\csc x\right) = \lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right) = \lim_{x\to 0}\frac{\sin x - x}{x\sin x}$

$$\overset{\text{H}}{=} \lim_{x\to 0}\frac{\cos x - 1}{x\cos x + \sin x} \overset{\text{H}}{=} \lim_{x\to 0}\frac{-\sin x}{2\cos x - x\sin x} = \frac{0}{2} = 0$$

28. $\lim_{x\to 0}(\csc x - \cot x) = \lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right) = \lim_{x\to 0}\frac{1-\cos x}{\sin x} \overset{\text{H}}{=} \lim_{x\to 0}\frac{\sin x}{\cos x} = 0$

29. As $x\to\infty$, $1/x\to 0$, and $e^{1/x}\to 1$. So the limit has the form $\infty-\infty$ and we will change the form to a product by factoring out x.

$$\lim_{x\to\infty}\left(xe^{1/x}-x\right) = \lim_{x\to\infty}x\left(e^{1/x}-1\right) = \lim_{x\to\infty}\frac{e^{1/x}-1}{1/x} \overset{\text{H}}{=} \lim_{x\to\infty}\frac{e^{1/x}(-1/x^2)}{-1/x^2} = \lim_{x\to\infty}e^{1/x} = e^0 = 1$$

30. $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) = \lim_{x\to 1}\frac{x-1-\ln x}{(x-1)\ln x} \overset{\text{H}}{=} \lim_{x\to 1}\frac{1-1/x}{(x-1)(1/x)+\ln x}\cdot\frac{x}{x}$

$$= \lim_{x\to 1}\frac{x-1}{x-1+x\ln x} \overset{\text{H}}{=} \lim_{x\to 1}\frac{1}{1+1+\ln x} = \frac{1}{2+0} = \frac{1}{2}$$