51. $\lim\limits_{x\to\infty} \dfrac{e^x}{x^n} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{nx^{n-1}} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{n(n-1)x^{n-2}} \overset{\text{H}}{=} \cdots \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{e^x}{n!} = \infty$

52. $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^p} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{1/x}{px^{p-1}} = \lim\limits_{x\to\infty} \dfrac{1}{px^p} = 0$ since $p > 0$.

54. (a) $\lim\limits_{t\to\infty} v = \lim\limits_{t\to\infty} \dfrac{mg}{c}\left(1 - e^{-ct/m}\right) = \dfrac{mg}{c} \lim\limits_{t\to\infty} \left(1 - e^{-ct/m}\right)$

$= \dfrac{mg}{c}(1 - 0)$ [because $-ct/m \to -\infty$ as $t \to \infty$] $= \dfrac{mg}{c}$,

which is the speed the object approaches as time goes on, the so-called limiting velocity.

(b) $\lim\limits_{m\to\infty} v = \lim\limits_{m\to\infty} \dfrac{mg}{c}\left(1 - e^{-ct/m}\right) = \dfrac{g}{c} \lim\limits_{m\to\infty} \dfrac{1 - e^{-ct/m}}{1/m} \overset{\text{H}}{=} \dfrac{g}{c} \lim\limits_{m\to\infty} \dfrac{-e^{-ct/m}\left(ct/m^2\right)}{-1/m^2}$

$= \dfrac{g}{c}(ct) \lim\limits_{m\to\infty} e^{-ct/m} = gt(1)$ [because $-ct/m \to 0$ as $m \to \infty$] $= gt$.

The speed of a very heavy falling object is approximately proportional to the elapsed time—it doesn't depend on the mass.