STEWART
5.10

1. Explain why each of the following integrals is improper.

(a) $\int_1^\infty x^4 e^{-x^4}\,dx$ (b) $\int_0^{\pi/2} \sec x\,dx$

(c) $\int_0^2 \frac{x}{x^2 - 5x + 6}\,dx$ (d) $\int_{-\infty}^0 \frac{1}{x^2 + 5}\,dx$

2. Which of the following integrals are improper? Why?

(a) $\int_1^2 \frac{1}{2x - 1}\,dx$ (b) $\int_0^1 \frac{1}{2x - 1}\,dx$

(c) $\int_{-\infty}^\infty \frac{\sin x}{1 + x^2}\,dx$ (d) $\int_1^2 \ln(x - 1)\,dx$

3. Find the area under the curve $y = 1/x^3$ from $x = 1$ to $x = t$ and evaluate it for $t = 10$, 100, and 1000. Then find the total area under this curve for $x \geq 1$.

4. (a) Graph the functions $f(x) = 1/x^{1.1}$ and $g(x) = 1/x^{0.9}$ in the viewing rectangles $[0, 10]$ by $[0, 1]$ and $[0, 100]$ by $[0, 1]$.
(b) Find the areas under the graphs of f and g from $x = 1$ to $x = t$ and evaluate for $t = 10$, 100, 10^4, 10^6, 10^{10}, and 10^{20}.
(c) Find the total area under each curve for $x \geq 1$, if it exists.

5–32 ■ Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

5. $\int_1^\infty \frac{1}{(3x + 1)^2}\,dx$

6. $\int_2^\infty \frac{1}{(x + 3)^{3/2}}\,dx$

7. $\int_0^\infty e^{-x}\,dx$

8. $\int_{-\infty}^0 \frac{1}{2x - 5}\,dx$

9. $\int_{-\infty}^{-1} \frac{1}{\sqrt{2 - w}}\,dw$

10. $\int_{-\infty}^{-1} e^{-2t}\,dt$

11. $\int_{-\infty}^\infty x^3\,dx$

12. $\int_{-\infty}^\infty (2 - v^4)\,dv$

13. $\int_{-\infty}^\infty xe^{-x^2}\,dx$

14. $\int_{-\infty}^\infty x^2 e^{-x^3}\,dx$

15. $\int_0^\infty \cos x\,dx$

16. $\int_{-\infty}^{\pi/2} \sin 2\theta\,d\theta$

17. $\int_{-\infty}^1 xe^{2x}\,dx$

18. $\int_0^\infty xe^{-x}\,dx$

19. $\int_1^\infty \frac{\ln x}{x}\,dx$

20. $\int_{-\infty}^\infty \frac{1}{r^2 + 4}\,dr$

21. $\int_1^\infty \frac{\ln x}{x^2}\,dx$

22. $\int_1^\infty \frac{\ln x}{x^3}\,dx$

23. $\int_0^3 \frac{1}{\sqrt{x}}\,dx$

24. $\int_0^3 \frac{1}{x\sqrt{x}}\,dx$

25. $\int_{-1}^0 \frac{1}{x^2}\,dx$

26. $\int_1^9 \frac{1}{\sqrt[3]{x - 9}}\,dx$

27. $\int_0^{\pi/4} \csc^2 t\,dt$

28. $\int_0^1 \frac{1}{4y - 1}\,dy$

29. $\int_{-2}^3 \frac{1}{x^4}\,dx$

30. $\int_0^4 \frac{1}{x^2 + x - 6}\,dx$

31. $\int_0^2 z^2 \ln z\,dz$

32. $\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx$

47. The integral

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1 + x)}\,dx$$

is improper for two reasons: the interval $[0, \infty)$ is infinite and the integrand has an infinite discontinuity at 0. Evaluate it by expressing it as a sum of improper integrals of Type 2 and Type 1 as follows:

$$\int_0^\infty \frac{1}{\sqrt{x}\,(1 + x)}\,dx = \int_0^1 \frac{1}{\sqrt{x}\,(1 + x)}\,dx + \int_1^\infty \frac{1}{\sqrt{x}\,(1 + x)}\,dx$$

48. Evaluate

$$\int_2^\infty \frac{1}{x\sqrt{x^2 - 4}}\,dx$$

by the same method as in Exercise 47.

49. Find the values of p for which the integral $\int_0^1 (1/x^p)\,dx$ converges and evaluate the integral for those values of p.

50. (a) Evaluate the integral $\int_0^\infty x^n e^{-x}\,dx$ for $n = 0$, 1, 2, and 3.
(b) Guess the value of $\int_0^\infty x^n e^{-x}\,dx$ when n is an arbitrary positive integer.
(c) Prove your guess using mathematical induction.

51. (a) Show that $\int_{-\infty}^\infty x\,dx$ is divergent.
(b) Show that

$$\lim_{t \to \infty} \int_{-t}^t x\,dx = 0$$

This shows that we can't define

$$\int_{-\infty}^\infty f(x)\,dx = \lim_{t \to \infty} \int_{-t}^t f(x)\,dx$$

52. If $\int_{-\infty}^\infty f(x)\,dx$ is convergent and a and b are real numbers, show that

$$\int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx = \int_{-\infty}^b f(x)\,dx + \int_b^\infty f(x)\,dx$$