5. $I = \int_1^\infty \frac{1}{(3x+1)^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{(3x+1)^2}\,dx$. Now

$$\int \frac{1}{(3x+1)^2}\,dx = \frac{1}{3}\int \frac{1}{u^2}\,du \quad [u = 3x+1,\, du = 3\,dx]$$

$$= -\frac{1}{3u} + C = -\frac{1}{3(3x+1)} + C,$$

so $I = \lim_{t\to\infty} \left[-\frac{1}{3(3x+1)}\right]_1^t = \lim_{t\to\infty} \left[-\frac{1}{3(3t+1)} + \frac{1}{12}\right] = 0 + \frac{1}{12} = \frac{1}{12}$. Convergent

6. $\int_2^\infty \frac{dx}{(x+3)^{3/2}} = \lim_{t\to\infty} \int_2^t \frac{dx}{(x+3)^{3/2}} = \lim_{t\to\infty} \left[\frac{-2}{\sqrt{x+3}}\right]_2^t = \lim_{t\to\infty} \left[\frac{-2}{\sqrt{t+3}} + \frac{2}{\sqrt{5}}\right] = \frac{2}{\sqrt{5}}$. Convergent

7. $\int_0^\infty e^{-x}\,dx = \lim_{t\to\infty} \int_0^t e^{-x}\,dx = \lim_{t\to\infty} \left[-e^{-x}\right]_0^t = \lim_{t\to\infty} \left(-e^{-t}+1\right) = 1$. Convergent

8. $\int_{-\infty}^0 \frac{1}{2x-5}\,dx = \lim_{t\to-\infty} \int_t^0 \frac{1}{2x-5}\,dx = \lim_{t\to-\infty} \left[\frac{1}{2}\ln|2x-5|\right]_t^0 = \lim_{t\to\infty} \left[\frac{1}{2}\ln 5 - \frac{1}{2}\ln|2t-5|\right] = -\infty$.
Divergent

9. $\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}}\,dw = \lim_{t\to-\infty} \int_t^{-1} \frac{1}{\sqrt{2-w}}\,dw = \lim_{t\to-\infty} \left[-2\sqrt{2-w}\right]_t^{-1}$ $[u = 2-w,\, du = -dw]$

$= \lim_{t\to-\infty} \left[-2\sqrt{3} + 2\sqrt{2-t}\right] = \infty$. Divergent

10. $\int_{-\infty}^{-1} e^{-2t}\,dt = \lim_{x\to-\infty} \int_x^{-1} e^{-2t}\,dt = \lim_{x\to-\infty} \left[-\frac{1}{2}e^{-2t}\right]_x^{-1} = \lim_{x\to-\infty} \left[-\frac{1}{2}e^2 + \frac{1}{2}e^{-2x}\right] = \infty$. Divergent

11. $I = \int_{-\infty}^\infty x^3\,dx = I_1 + I_2 = \int_{-\infty}^0 x^3\,dx + \int_0^\infty x^3\,dx$, but $I_1 = \lim_{t\to-\infty} \left[\frac{1}{4}x^4\right]_t^0 = \lim_{t\to-\infty} \left(-\frac{1}{4}t^4\right) = -\infty$. Since I_1 is divergent, I is divergent, and there is no need to evaluate I_2. Divergent

12. $I = \int_{-\infty}^\infty (2-v^4)\,dv = I_1 + I_2 = \int_{-\infty}^0 (2-v^4)\,dv + \int_0^\infty (2-v^4)\,dv$, but
$I_1 = \lim_{t\to-\infty} \left[2v - \frac{1}{5}v^5\right]_t^0 = \lim_{t\to-\infty} \left(-2t + \frac{1}{5}t^5\right) = -\infty$. Since I_1 is divergent, I is divergent, and there is no need to evaluate I_2. Divergent

13. $\int_{-\infty}^\infty xe^{-x^2}\,dx = \int_{-\infty}^0 xe^{-x^2}\,dx + \int_0^\infty xe^{-x^2}\,dx$.

$\int_{-\infty}^0 xe^{-x^2}\,dx = \lim_{t\to-\infty} \left(-\frac{1}{2}\right)\left[e^{-x^2}\right]_t^0 = \lim_{t\to-\infty} \left(-\frac{1}{2}\right)\left(1 - e^{-t^2}\right) = -\frac{1}{2}\cdot 1 = -\frac{1}{2}$, and

$\int_0^\infty xe^{-x^2}\,dx = \lim_{t\to\infty} \left(-\frac{1}{2}\right)\left[e^{-x^2}\right]_0^t = \lim_{t\to\infty} \left(-\frac{1}{2}\right)\left(e^{-t^2} - 1\right) = -\frac{1}{2}\cdot(-1) = \frac{1}{2}$.

Therefore, $\int_{-\infty}^\infty xe^{-x^2}\,dx = -\frac{1}{2} + \frac{1}{2} = 0$. Convergent

14. $\int_{-\infty}^\infty x^2e^{-x^3}\,dx = \int_{-\infty}^0 x^2e^{-x^3}\,dx + \int_0^\infty x^2e^{-x^3}\,dx$, and

$\int_{-\infty}^0 x^2e^{-x^3}\,dx = \lim_{t\to-\infty} \left[-\frac{1}{3}e^{-x^3}\right]_t^0 = -\frac{1}{3} + \frac{1}{3}\left(\lim_{t\to-\infty} e^{-t^3}\right) = \infty$. Divergent

15. $\int_0^\infty \cos x\,dx = \lim_{t\to\infty} \left[\sin x\right]_0^t = \lim_{t\to\infty} \sin t$, which does not exist. Divergent

16. $\int_{-\infty}^{\pi/2} \sin 2\theta\,d\theta = \lim_{t\to-\infty} \int_t^{\pi/2} \sin 2\theta\,d\theta = \lim_{t\to-\infty} \left[-\frac{1}{2}\cos 2\theta\right]_t^{\pi/2} = \lim_{t\to-\infty} \left(\frac{1}{2} + \frac{1}{2}\cos 2t\right)$. This limit does not exist, so the integral is divergent.