17. $\int_{-\infty}^{1} xe^{2x}\,dx = \lim\limits_{t\to-\infty} \int_t^1 xe^{2x}\,dx = \lim\limits_{t\to-\infty} \left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right]_t^1$ (by parts with $u = x$ and $dv = e^{2x}\,dx$)

$$= \lim_{t\to-\infty} \left[\tfrac{1}{2}e^2 - \tfrac{1}{4}e^2 - \tfrac{1}{2}te^{2t} + \tfrac{1}{4}e^{2t}\right] = \tfrac{1}{4}e^2 - 0 + 0 = \tfrac{1}{4}e^2$$

since $\lim\limits_{t\to-\infty} te^{2t} = \lim\limits_{t\to-\infty} \dfrac{t}{e^{-2t}} \overset{\text{H}}{=} \lim\limits_{t\to-\infty} \dfrac{1}{-2e^{-2t}} = \lim\limits_{t\to-\infty} -\tfrac{1}{2}e^{2t} = 0.$ Convergent

18. $\int_0^\infty xe^{-x}\,dx = \lim\limits_{t\to\infty} \left[-xe^{-x} - e^{-x}\right]_0^t$ (by parts with $u = x$ and $dv = e^{-x}\,dx$)

$$= \lim_{t\to\infty} \left[-te^{-t} - e^{-t} + 1\right] = 0 - 0 + 1 = 1,$$

since $\lim\limits_{t\to\infty} (te^{-t}) = \lim\limits_{t\to\infty} \dfrac{t}{e^t} \overset{\text{H}}{=} \lim\limits_{t\to\infty} \dfrac{1}{e^t} = 0.$ Convergent

19. $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{t\to\infty} \left[\frac{(\ln x)^2}{2}\right]_1^t$ (by substitution with $u = \ln x$, $du = dx/x$) $= \displaystyle\lim_{t\to\infty} \frac{(\ln t)^2}{2} = \infty.$ Divergent

20. Since $f(r) = \dfrac{1}{r^2+4}$ is even,

$$I = \int_{-\infty}^{\infty} f(r)\,dr = 2\int_0^\infty f(r)\,dr = 2\lim_{t\to\infty} \int_0^t \frac{1}{r^2+4}\,dr = 2\lim_{t\to\infty} \left[\frac{1}{2}\arctan\frac{r}{2}\right]_0^t$$

$$= \lim_{t\to\infty} \left(\arctan\frac{t}{2} - 0\right) = \frac{\pi}{2}. \quad \text{Convergent}$$

21. Integrate by parts with $u = \ln x$, $dv = dx/x^2$ $\Rightarrow$ $du = dx/x$, $v = -1/x$.

$$\int_1^\infty \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x^2}\,dx = \lim_{t\to\infty} \left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^t = \lim_{t\to\infty} \left(-\frac{\ln t}{t} - \frac{1}{t} + 0 + 1\right)$$

$$= -0 - 0 + 0 + 1 = 1$$

since $\lim\limits_{t\to\infty} \dfrac{\ln t}{t} \overset{\text{H}}{=} \lim\limits_{t\to\infty} \dfrac{1/t}{1} = 0.$ Convergent

22. Integrate by parts with $u = \ln x$, $dv = dx/x^3$ $\Rightarrow$ $du = dx/x$, $v = -1/\left(2x^2\right)$.

$$\int_1^\infty \frac{\ln x}{x^3}\,dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x^3}\,dx = \lim_{t\to\infty} \left(\left[-\frac{1}{2x^2}\ln x\right]_1^t + \frac{1}{2}\int_1^t \frac{1}{x^3}\,dx\right)$$

$$= \lim_{t\to\infty} \left(-\frac{1}{2}\frac{\ln t}{t^2} + 0 - \frac{1}{4t^2} + \frac{1}{4}\right) = \frac{1}{4}$$

since $\lim\limits_{t\to\infty} \dfrac{\ln t}{t^2} \overset{\text{H}}{=} \lim\limits_{t\to\infty} \dfrac{1/t}{2t} = \lim\limits_{t\to\infty} \dfrac{1}{2t^2} = 0.$ Convergent

23. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{\sqrt{x}} = \lim_{t\to0^+} \int_t^3 \frac{dx}{\sqrt{x}} = \lim_{t\to0^+} \left[2\sqrt{x}\right]_t^3 = \lim_{t\to0^+} \left(2\sqrt{3} - 2\sqrt{t}\right) = 2\sqrt{3}. \quad \text{Convergent}$$

24. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{x\sqrt{x}} = \lim_{t\to0^+} \int_t^3 \frac{dx}{x^{3/2}} = \lim_{t\to0^+} \left[\frac{-2}{\sqrt{x}}\right]_t^3 = \frac{-2}{\sqrt{3}} + \lim_{t\to0^+} \frac{2}{\sqrt{t}} = \infty. \quad \text{Divergent}$$

25. There is an infinite discontinuity at the right endpoint of $[-1, 0]$.

$$\int_{-1}^0 \frac{dx}{x^2} = \lim_{t\to0^-} \int_{-1}^t \frac{dx}{x^2} = \lim_{t\to0^-} \left[\frac{-1}{x}\right]_{-1}^t = \lim_{t\to0^-} \left[-\frac{1}{t} + \frac{1}{-1}\right] = \infty. \quad \text{Divergent}$$