26. $\displaystyle\int_1^9 \frac{dx}{\sqrt[3]{x-9}} = \lim_{t\to 9^-}\int_1^t \frac{dx}{\sqrt[3]{x-9}} = \lim_{t\to 9^-}\left[\tfrac{3}{2}(x-9)^{2/3}\right]_1^t = \lim_{t\to 9^-}\left[\tfrac{3}{2}(t-9)^{2/3} - \tfrac{3}{2}(4)\right] = 0 - 6 = -6.$
Convergent

27. $\int_0^{\pi/4} \csc^2 t\,dt = \lim_{s\to 0^+}\int_s^{\pi/4} \csc^2 t\,dt = \lim_{s\to 0^+}\left[-\cot t\right]_s^{\pi/4} = \lim_{s\to 0^+}\left[-\cot\frac{\pi}{4} + \cot s\right] = \infty$. Divergent

28. $f(y) = 1/(4y-1)$ has an infinite discontinuity at $y = \frac{1}{4}$.

$$\int_{1/4}^1 \frac{1}{4y-1}\,dy = \lim_{t\to(1/4)^+}\int_t^1 \frac{1}{4y-1}\,dy = \lim_{t\to(1/4)^+}\left[\tfrac{1}{4}\ln|4y-1|\right]_t^1$$
$$= \lim_{t\to(1/4)^+}\left[\tfrac{1}{4}\ln 3 - \tfrac{1}{4}\ln(4t-1)\right] = \infty$$

so $\displaystyle\int_{1/4}^1 \frac{1}{4y-1}\,dy$ diverges, and hence, $\displaystyle\int_0^1 \frac{1}{4y-1}\,dy$ diverges. Divergent

29. $\displaystyle\int_{-2}^3 \frac{dx}{x^4} = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}$, but $\displaystyle\int_{-2}^0 \frac{dx}{x^4} = \lim_{t\to 0^-}\left[-\frac{x^{-3}}{3}\right]_{-2}^t = \lim_{t\to 0^-}\left[-\frac{1}{3t^3} - \frac{1}{24}\right] = \infty$. Divergent

30. $\displaystyle\int_0^4 \frac{dx}{x^2+x-6} = \int_0^4 \frac{dx}{(x+3)(x-2)} = \int_0^2 \frac{dx}{(x-2)(x+3)} + \int_2^4 \frac{dx}{(x-2)(x+3)}$, and

$$\int_0^2 \frac{dx}{(x-2)(x+3)} = \lim_{t\to 2^-}\int_0^t \left[\frac{1/5}{x-2} - \frac{1/5}{x+3}\right]dx \text{ (partial fractions) } = \lim_{t\to 2^-}\left[\frac{1}{5}\ln\left|\frac{x-2}{x+3}\right|\right]_0^t$$
$$= \lim_{t\to 2^-}\frac{1}{5}\left[\ln\left|\frac{t-2}{t+3}\right| - \ln\frac{2}{3}\right] = -\infty. \text{ Divergent}$$

31. $I = \int_0^2 z^2\ln z\,dz = \lim_{t\to 0^+}\int_t^2 z^2\ln z\,dz \overset{101}{=} \lim_{t\to 0^+}\left[\frac{z^3}{3^2}(3\ln z - 1)\right]_t^2$
$= \lim_{t\to 0^+}\left[\frac{8}{9}(3\ln 2 - 1) - \frac{1}{9}t^3(3\ln t - 1)\right] = \frac{8}{3}\ln 2 - \frac{8}{9} - \frac{1}{9}\lim_{t\to 0^+}\left[t^3(3\ln t - 1)\right] = \frac{8}{3}\ln 2 - \frac{8}{9} - \frac{1}{9}L.$
Now $L = \lim_{t\to 0^+}\left[t^3(3\ln t - 1)\right] = \lim_{t\to 0^+}\dfrac{3\ln t - 1}{t^{-3}} \overset{H}{=} \lim_{t\to 0^+}\dfrac{3/t}{-3/t^4} = \lim_{t\to 0^+}(-t^3) = 0$. Thus, $L = 0$ and
$I = \frac{8}{3}\ln 2 - \frac{8}{9}$. Convergent

32. Integrate by parts with $u = \ln x$, $dv = dx/\sqrt{x}$ $\Rightarrow$ $du = dx/x$, $v = 2\sqrt{x}$.

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{t\to 0^+}\int_t^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{t\to 0^+}\left(\left[2\sqrt{x}\ln x\right]_t^1 - 2\int_t^1 \frac{dx}{\sqrt{x}}\right) = \lim_{t\to 0^+}\left(-2\sqrt{t}\ln t - 4\left[\sqrt{x}\right]_t^1\right)$$
$$= \lim_{t\to 0^+}\left(-2\sqrt{t}\ln t - 4 + 4\sqrt{t}\right) = -4$$

since $\lim_{t\to 0^+}\sqrt{t}\ln t = \lim_{t\to 0^+}\dfrac{\ln t}{t^{-1/2}} \overset{H}{=} \lim_{t\to 0^+}\dfrac{1/t}{-t^{-3/2}/2} = \lim_{t\to 0^+}(-2\sqrt{t}) = 0$. Convergent