

2. (a) Since  $y = 1/(2x - 1)$  is defined and continuous on  $[1, 2]$ , the integral is proper.

(b) Since  $y = \frac{1}{2x - 1}$  has an infinite discontinuity at  $x = \frac{1}{2}$ ,  $\int_0^1 \frac{1}{2x - 1} dx$  is a Type II improper integral.

(c) Since  $\int_{-\infty}^{\infty} \frac{\sin x}{1 + x^2} dx$  has an infinite interval of integration, it is an improper integral of Type I.

(d) Since  $y = \ln(x - 1)$  has an infinite discontinuity at  $x = 1$ ,  $\int_1^2 \ln(x - 1) dx$  is a Type II improper integral.

3. The area under the graph of  $y = 1/x^3 = x^{-3}$  between  $x = 1$  and  $x = t$  is

$A(t) = \int_1^t x^{-3} dx = \left[-\frac{1}{2}x^{-2}\right]_1^t = -\frac{1}{2}t^{-2} - \left(-\frac{1}{2}\right) = \frac{1}{2} - 1/(2t^2)$ . So the area for  $1 \leq x \leq 10$  is

$A(10) = 0.5 - 0.005 = 0.495$ , the area for  $1 \leq x \leq 100$  is  $A(100) = 0.5 - 0.00005 = 0.49995$ , and the area for  $1 \leq x \leq 1000$  is  $A(1000) = 0.5 - 0.0000005 = 0.4999995$ . The total area under the curve for  $x \geq 1$  is

$\lim_{t \rightarrow \infty} A(t) = \lim_{t \rightarrow \infty} \left[\frac{1}{2} - 1/(2t^2)\right] = \frac{1}{2}$ .