23. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{\sqrt{x}} = \lim_{t\to 0^+} \int_t^3 \frac{dx}{\sqrt{x}} = \lim_{t\to 0^+} \left[2\sqrt{x}\right]_t^3 = \lim_{t\to 0^+} \left(2\sqrt{3} - 2\sqrt{t}\right) = 2\sqrt{3}.$$ Convergent

24. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{x\sqrt{x}} = \lim_{t\to 0^+} \int_t^3 \frac{dx}{x^{3/2}} = \lim_{t\to 0^+} \left[\frac{-2}{\sqrt{x}}\right]_t^3 = \frac{-2}{\sqrt{3}} + \lim_{t\to 0^+} \frac{2}{\sqrt{t}} = \infty.$$ Divergent

25. There is an infinite discontinuity at the right endpoint of $[-1, 0]$.

$$\int_{-1}^0 \frac{dx}{x^2} = \lim_{t\to 0^-} \int_{-1}^t \frac{dx}{x^2} = \lim_{t\to 0^-} \left[\frac{-1}{x}\right]_{-1}^t = \lim_{t\to 0^-} \left[-\frac{1}{t} + \frac{1}{-1}\right] = \infty.$$ Divergent

26. $\int_1^9 \frac{dx}{\sqrt[3]{x-9}} = \lim_{t\to 9^-} \int_1^t \frac{dx}{\sqrt[3]{x-9}} = \lim_{t\to 9^-} \left[\frac{3}{2}(x-9)^{2/3}\right]_1^t = \lim_{t\to 9^-} \left[\frac{3}{2}(t-9)^{2/3} - \frac{3}{2}(4)\right] = 0 - 6 = -6.$
Convergent

27. $\int_0^{\pi/4} \csc^2 t\, dt = \lim_{s\to 0+} \int_s^{\pi/4} \csc^2 t\, dt = \lim_{s\to 0+} \left[-\cot t\right]_s^{\pi/4} = \lim_{s\to 0+} \left[-\cot\frac{\pi}{4} + \cot s\right] = \infty.$ Divergent

28. $f(y) = 1/(4y-1)$ has an infinite discontinuity at $y = \frac{1}{4}$.

$$\int_{1/4}^1 \frac{1}{4y-1}\, dy = \lim_{t\to(1/4)+} \int_t^1 \frac{1}{4y-1}\, dy = \lim_{t\to(1/4)+} \left[\tfrac{1}{4}\ln|4y-1|\right]_t^1$$

$$= \lim_{t\to(1/4)+} \left[\tfrac{1}{4}\ln 3 - \tfrac{1}{4}\ln(4t-1)\right] = \infty$$

so $\int_{1/4}^1 \frac{1}{4y-1}\, dy$ diverges, and hence, $\int_0^1 \frac{1}{4y-1}\, dy$ diverges. Divergent

29. $\int_{-2}^3 \frac{dx}{x^4} = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}$, but $\int_{-2}^0 \frac{dx}{x^4} = \lim_{t\to 0^-} \left[-\frac{x^{-3}}{3}\right]_{-2}^t = \lim_{t\to 0^-} \left[-\frac{1}{3t^3} - \frac{1}{24}\right] = \infty.$ Divergent