

$$26. \int_1^9 \frac{dx}{\sqrt[3]{x-9}} = \lim_{t \rightarrow 9^-} \int_1^t \frac{dx}{\sqrt[3]{x-9}} = \lim_{t \rightarrow 9^-} \left[\frac{3}{2}(x-9)^{2/3} \right]_1^t = \lim_{t \rightarrow 9^-} \left[\frac{3}{2}(t-9)^{2/3} - \frac{3}{2}(4) \right] = 0 - 6 = -6.$$

Convergent

$$27. \int_0^{\pi/4} \csc^2 t \, dt = \lim_{s \rightarrow 0^+} \int_s^{\pi/4} \csc^2 t \, dt = \lim_{s \rightarrow 0^+} [-\cot t]_s^{\pi/4} = \lim_{s \rightarrow 0^+} [-\cot \frac{\pi}{4} + \cot s] = \infty. \text{ Divergent}$$

28. $f(y) = 1/(4y - 1)$ has an infinite discontinuity at $y = \frac{1}{4}$.

$$\begin{aligned} \int_{1/4}^1 \frac{1}{4y-1} dy &= \lim_{t \rightarrow (1/4)^+} \int_t^1 \frac{1}{4y-1} dy = \lim_{t \rightarrow (1/4)^+} \left[\frac{1}{4} \ln |4y-1| \right]_t^1 \\ &= \lim_{t \rightarrow (1/4)^+} \left[\frac{1}{4} \ln 3 - \frac{1}{4} \ln(4t-1) \right] = \infty \end{aligned}$$

so $\int_{1/4}^1 \frac{1}{4y-1} dy$ diverges, and hence, $\int_0^1 \frac{1}{4y-1} dy$ diverges. Divergent

$$29. \int_{-2}^3 \frac{dx}{x^4} = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}, \text{ but } \int_{-2}^0 \frac{dx}{x^4} = \lim_{t \rightarrow 0^-} \left[-\frac{x^{-3}}{3} \right]_{-2}^t = \lim_{t \rightarrow 0^-} \left[-\frac{1}{3t^3} - \frac{1}{24} \right] = \infty. \text{ Divergent}$$