

49. If $p = 1$, then $\int_0^1 \frac{dx}{x^p} = \lim_{t \rightarrow 0^+} \int_t^1 \frac{dx}{x} = \lim_{t \rightarrow 0^+} [\ln x]_t^1 = \infty$. Divergent.

If $p \neq 1$, then $\int_0^1 \frac{dx}{x^p} = \lim_{t \rightarrow 0^+} \int_t^1 \frac{dx}{x^p}$ (note that the integral is not improper if $p < 0$)

$$= \lim_{t \rightarrow 0^+} \left[\frac{x^{-p+1}}{-p+1} \right]_t^1 = \lim_{t \rightarrow 0^+} \frac{1}{1-p} \left[1 - \frac{1}{t^{p-1}} \right]$$

If $p > 1$, then $p - 1 > 0$, so $\frac{1}{t^{p-1}} \rightarrow \infty$ as $t \rightarrow 0^+$, and the integral diverges.

If $p < 1$, then $p - 1 < 0$, so $\frac{1}{t^{p-1}} \rightarrow 0$ as $t \rightarrow 0^+$ and $\int_0^1 \frac{dx}{x^p} = \frac{1}{1-p} \left[\lim_{t \rightarrow 0^+} (1 - t^{1-p}) \right] = \frac{1}{1-p}$.

Thus, the integral converges if and only if $p < 1$, and in that case its value is $\frac{1}{1-p}$.