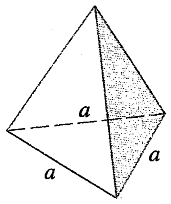
**25.** A pyramid with height $h$ and rectangular base with dimensions $b$ and $2b$

**26.** A pyramid with height $h$ and base an equilateral triangle with side $a$ (a tetrahedron)




**27.** A tetrahedron with three mutually perpendicular faces and three mutually perpendicular edges with lengths 3 cm, 4 cm, and 5 cm

**28.** The base of $S$ is a circular disk with radius $r$. Parallel cross-sections perpendicular to the base are squares.

**29.** The base of $S$ is an elliptical region with boundary curve $9x^2 + 4y^2 = 36$. Cross-sections perpendicular to the $x$-axis are isosceles right triangles with hypotenuse in the base.

**30.** The base of $S$ is the parabolic region $\{(x, y) \mid x^2 \leqslant y \leqslant 1\}$. Cross-sections perpendicular to the $y$-axis are equilateral triangles.

**31.** $S$ has the same base as in Exercise 30, but cross-sections perpendicular to the $y$-axis are squares.

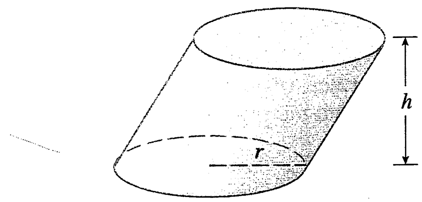
**32.** The base of $S$ is the triangular region with vertices $(0, 0)$, $(3, 0)$, and $(0, 2)$. Cross-sections perpendicular to the $y$-axis are semicircles.

**33.** $S$ has the same base as in Exercise 32 but cross-sections perpendicular to the $y$-axis are isosceles triangles with height equal to the base.

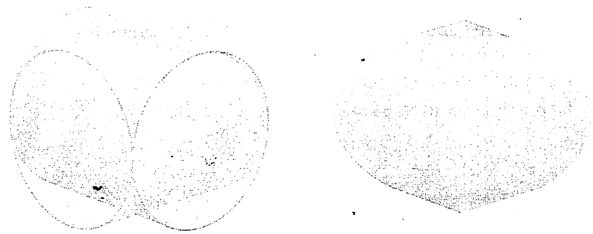
**34.** The base of $S$ is a circular disk with radius $r$. Parallel cross-sections perpendicular to the base are isosceles triangles with height $h$ and unequal side in the base.
(a) Set up an integral for the volume of $S$.
(b) By interpreting the integral as an area, find the volume of $S$.

**36.** A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30° along a diameter of the cylinder. Find the volume of the wedge.

**37.** (a) Cavalieri's Principle states that if a family of parallel planes gives equal cross-sectional areas for two solids $S_1$ and $S_2$, then the volumes of $S_1$ and $S_2$ are equal. Prove this principle.
(b) Use Cavalieri's Principle to find the volume of the oblique cylinder shown in the figure.




**38.** Find the volume common to two circular cylinders, each with radius $r$, if the axes of the cylinders intersect at right angles.



**39.** Find the volume common to two spheres, each with radius $r$, if the center of each sphere lies on the surface of the other sphere.

**40.** A bowl is shaped like a hemisphere with diameter 30 cm. A ball with diameter 10 cm is placed in the bowl and water is poured into the bowl to a depth of $h$ centimeters. Find the volume of water in the bowl.

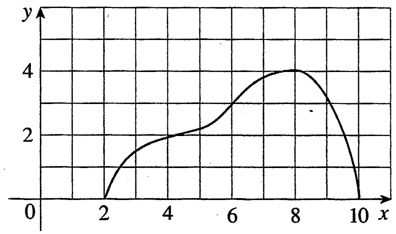
**41.** A hole of radius $r$ is bored through a cylinder of radius $R > r$ at right angles to the axis of the cylinder. Set up, but do not evaluate, an integral for the volume cut out.

**42.** A hole of radius $r$ is bored through the center of a sphere of radius $R > r$. Find the volume of the remaining portion of the sphere.

**43.** Let $S$ be the solid obtained by rotating about the $y$-axis the region bounded by $y = x(x - 1)^2$ and $y = 0$. Explain why it is awkward to use slicing to find the volume $V$ of $S$. Then find $V$ using cylindrical shells.

**44.** Let $S$ be the solid obtained by rotating the region under the curve $y = \sin(x^2)$ from 0 to $\sqrt{\pi}$ about the $y$-axis. Sketch a typical cylindrical shell and find its circumference and height. Use shells to find the volume of $S$. Do you think this method is preferable to slicing? Explain.

**45.** If the region shown in the figure is rotated about the $y$-axis to form a solid, use Simpson's Rule with $n = 8$ to estimate the volume of the solid.




**46.** Let $V$ be the volume of the solid obtained by rotating about the $y$-axis the region bounded by $y = x$ and $y = x^2$. Find $V$ both by slicing and by cylindrical shells. In both cases draw a diagram to explain your method.

**47.** Use cylindrical shells to find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$. Sketch the region and a typical shell. Explain why this method is preferable to slicing.

**48.** Suppose you make napkin rings by drilling holes with different diameters through two wooden balls (which also have different diameters). You discover that both napkin rings have the same height $h$, as shown in the figure.
(a) Guess which ring has more wood in it.
(b) Check your guess: Use cylindrical shells to compute the volume of a napkin ring created by drilling a hole with radius $r$ through the center of a sphere of radius $R$ and express the answer in terms of $h$.