MATHEMATICAL ASSOCIATION OF AMERICA

American Mathematics Competitions

12th Annual

AMC 10 B

American Mathematics Contest 10 B

Wednesday, February 23, 2011

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form.
8. When your proctor gives the signal, begin working on the problems. You will have **75 minutes** to complete the test.
9. When you finish the exam, *sign your name* in the space provided on the Answer Form.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score 120 or above or finish in the top 2.5% on this AMC 10 will be invited to take the 29th annual American Invitational Mathematics Examination (AIME) on Thursday, March 17, 2011 or Wednesday, March 30, 2011. More details about the AIME and other information are on the back page of this test booklet.

2011

AMC 10 B

DO NOT OPEN UNTIL WEDNESDAY, FEBRUARY 23, 2011

****Administration On An Earlier Date Will Disqualify Your School's Results****

1. All information (Rules and Instructions) needed to administer this exam is contained in the TEACHERS' MANUAL, which is outside of this package. PLEASE READ THE MANUAL BEFORE FEBRUARY 23, 2011. Nothing is needed from inside this package until February 23.
2. Your PRINCIPAL or VICE-PRINCIPAL must verify on the AMC 10 CERTIFICATION FORM (found in the Teachers' Manual) that you followed all rules associated with the conduct of the exam.
3. The Answer Forms must be mailed by trackable mail to the AMC office no later than 24 hours following the exam.
4. *The publication, reproduction or communication of the problems or solutions of this test during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination at any time via copier, telephone, e-mail, internet or media of any type is a violation of the competition rules.*

1. What is

$$\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}\,?$$

(A) -1 **(B)** $\frac{5}{36}$ **(C)** $\frac{7}{12}$ **(D)** $\frac{147}{60}$ **(E)** $\frac{43}{3}$

2. Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

(A) 80 **(B)** 82 **(C)** 85 **(D)** 90 **(E)** 95

3. At a store, when a length is reported as x inches that means the length is at least $x - 0.5$ inches and at most $x + 0.5$ inches. Suppose the dimensions of a rectangular tile are reported as 2 inches by 3 inches. In square inches, what is the minimum area for the rectangle?

(A) 3.75 **(B)** 4.5 **(C)** 5 **(D)** 6 **(E)** 8.75

4. LeRoy and Bernardo went on a week-long trip together and agreed to share the costs equally. Over the week, each of them paid for various joint expenses such as gasoline and car rental. At the end of the trip it turned out that LeRoy had paid A dollars and Bernardo had paid B dollars, where $A < B$. How many dollars must LeRoy give to Bernardo so that they share the costs equally?

(A) $\frac{A+B}{2}$ **(B)** $\frac{A-B}{2}$ **(C)** $\frac{B-A}{2}$ **(D)** $B-A$ **(E)** $A+B$

5. In multiplying two positive integers a and b, Ron reversed the digits of the two-digit number a. His erroneous product was 161. What is the correct value of the product of a and b?

(A) 116 **(B)** 161 **(C)** 204 **(D)** 214 **(E)** 224

6. On Halloween Casper ate $\frac{1}{3}$ of his candies and then gave 2 candies to his brother. The next day he ate $\frac{1}{3}$ of his remaining candies and then gave 4 candies to his sister. On the third day he ate his final 8 candies. How many candies did Casper have at the beginning?

(A) 30 **(B)** 39 **(C)** 48 **(D)** 57 **(E)** 66

7. The sum of two angles of a triangle is $\frac{6}{5}$ of a right angle, and one of these two angles is 30° larger than the other. What is the degree measure of the largest angle in the triangle?

(A) 69 **(B)** 72 **(C)** 90 **(D)** 102 **(E)** 108

8. At a certain beach if it is at least 80° F and sunny, then the beach will be crowded. On June 10 the beach was not crowded. What can be concluded about the weather conditions on June 10?

(A) The temperature was cooler than 80° F and it was not sunny.

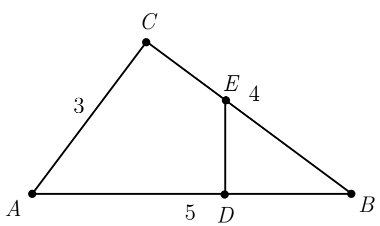
(B) The temperature was cooler than 80° F or it was not sunny.

(C) If the temperature was at least 80° F, then it was sunny.

(D) If the temperature was cooler than 80° F, then it was sunny.

(E) If the temperature was cooler than 80° F, then it was not sunny.

9. The area of $\triangle EBD$ is one third of the area of $3-4-5$ $\triangle ABC$. Segment DE is perpendicular to segment AB. What is BD?



(A) $\frac{4}{3}$ **(B)** $\sqrt{5}$ **(C)** $\frac{9}{4}$ **(D)** $\frac{4\sqrt{3}}{3}$ **(E)** $\frac{5}{2}$

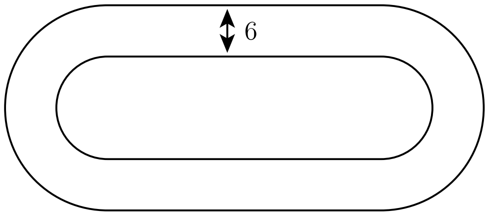
10. Consider the set of numbers $\{1, 10, 10^2, 10^3, \ldots, 10^{10}\}$. The ratio of the largest element of the set to the sum of the other ten elements of the set is closest to which integer?

(A) 1 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 101

11. There are 52 people in a room. What is the largest value of n such that the statement "At least n people in this room have birthdays falling in the same month" is always true?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 12

12. Keiko walks once around a track at exactly the same constant speed every day. The sides of the track are straight, and the ends are semicircles. The track has width 6 meters, and it takes her 36 seconds longer to walk around the outside edge of the track than around the inside edge. What is Keiko's speed in meters per second?



(A) $\frac{\pi}{3}$ **(B)** $\frac{2\pi}{3}$ **(C)** π **(D)** $\frac{4\pi}{3}$ **(E)** $\frac{5\pi}{3}$

13. Two real numbers are selected independently at random from the interval $[-20, 10]$. What is the probability that the product of those numbers is greater than zero?

(A) $\frac{1}{9}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

14. A rectangular parking lot has a diagonal of 25 meters and an area of 168 square meters. In meters, what is the perimeter of the parking lot?

(A) 52 **(B)** 58 **(C)** 62 **(D)** 68 **(E)** 70

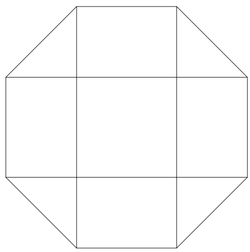
15. Let @ denote the "averaged with" operation: $a \,@\, b = \frac{a+b}{2}$. Which of the following distributive laws hold for all numbers x, y, and z?

$$\begin{aligned} \text{I.} \quad & x \,@\, (y+z) = (x \,@\, y) + (x \,@\, z) \\ \text{II.} \quad & x + (y \,@\, z) = (x+y) \,@\, (x+z) \\ \text{III.} \quad & x \,@\, (y \,@\, z) = (x \,@\, y) \,@\, (x \,@\, z) \end{aligned}$$

(A) I only **(B)** II only **(C)** III only **(D)** I and III only

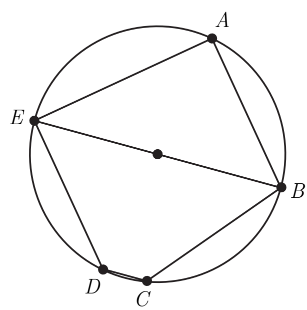
(E) II and III only

16. A dart board is a regular octagon divided into regions as shown. Suppose that a dart thrown at the board is equally likely to land anywhere on the board. What is the probability that the dart lands within the center square?



(A) $\frac{\sqrt{2}-1}{2}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{2-\sqrt{2}}{2}$ **(D)** $\frac{\sqrt{2}}{4}$ **(E)** $2-\sqrt{2}$

17. In the given circle, the diameter $\overline{EB}$ is parallel to $\overline{DC}$, and $\overline{AB}$ is parallel to $\overline{ED}$. The angles AEB and ABE are in the ratio $4:5$. What is the degree measure of angle BCD?



(A) 120 **(B)** 125 **(C)** 130 **(D)** 135 **(E)** 140

18. Rectangle $ABCD$ has $AB = 6$ and $BC = 3$. Point M is chosen on side AB so that $\angle AMD = \angle CMD$. What is the degree measure of $\angle AMD$?

(A) 15 **(B)** 30 **(C)** 45 **(D)** 60 **(E)** 75

19. What is the product of all the roots of the equation

$$\sqrt{5|x|+8} = \sqrt{x^2-16}.$$

(A) -64 **(B)** -24 **(C)** -9 **(D)** 24 **(E)** 576

20. Rhombus $ABCD$ has side length 2 and $\angle B = 120°$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $1+\frac{\sqrt{3}}{3}$ **(E)** 2

21. Brian writes down four integers $w > x > y > z$ whose sum is 44. The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9. What is the sum of the possible values for w?

(A) 16 **(B)** 31 **(C)** 48 **(D)** 62 **(E)** 93

22. A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

(A) $5\sqrt{2}-7$ **(B)** $7-4\sqrt{3}$ **(C)** $\frac{2\sqrt{2}}{27}$ **(D)** $\frac{\sqrt{2}}{9}$ **(E)** $\frac{\sqrt{3}}{9}$

23. What is the hundreds digit of 2011^{2011}?

(A) 1 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

24. A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \leq 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a?

(A) $\frac{51}{101}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{51}{100}$ **(D)** $\frac{52}{101}$ **(E)** $\frac{13}{25}$

25. Let T_1 be a triangle with sides 2011, 2012, and 2013. For $n \geq 1$, if $T_n = \triangle ABC$ and D, E, and F are the points of tangency of the incircle of $\triangle ABC$ to the sides AB, BC, and AC, respectively, then T_{n+1} is a triangle with side lengths AD, BE, and CF, if it exists. What is the perimeter of the last triangle in the sequence (T_n)?

(A) $\frac{1509}{8}$ **(B)** $\frac{1509}{32}$ **(C)** $\frac{1509}{64}$ **(D)** $\frac{1509}{128}$ **(E)** $\frac{1509}{256}$

MATHEMATICAL ASSOCIATION OF AMERICA

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WRITE TO US!

Correspondence about the problems and solutions for this AMC 10 and orders for publications should be addressed to:

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The problems and solutions for this AMC 10 were prepared by the MAA's Committee on the AMC 10 and AMC 12 under the direction of AMC 10 Subcommittee Chair:

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2011 AIME

The 29th annual AIME will be held on Thursday, March 17, with the alternate on Wednesday, March 30. It is a 15-question, 3-hour, integer-answer exam. You will be invited to participate only if you score 120 or above or finish in the top 2.5% of the AMC 10, or if you score 100 or above or finish in the top 5% of the AMC 12. Top-scoring students on the AMC 10/12/AIME will be selected to take the 40th Annual USA Mathematical Olympiad (USAMO) on April 27 - 28, 2011. The best way to prepare for the AIME and USAMO is to study previous exams. Copies may be ordered as indicated below.

PUBLICATIONS

A complete listing of current publications, with ordering instructions, is at our web site: amc.maa.org