

## R.4 Equations

1.  $.2m - .5 = .1m + .7$   
 $10(.2m - .5) = 10(.1m + .7)$   
 $2m - 5 = m + 7$   
 $m - 5 = 7$   
 $m = 12$

The solution is 12.

2.  $\frac{5}{6}k - 2k + \frac{1}{3} = \frac{2}{3}$

Multiply both sides of the equation by 6.

$$6\left(\frac{5}{6}k\right) - 6(2k) + 6\left(\frac{1}{3}\right) = 6\left(\frac{2}{3}\right)$$
$$5k - 12k + 2 = 4$$
$$-7k + 2 = 4$$
$$-7k = 2$$
$$k = -\frac{2}{7}$$

The solution is  $-\frac{2}{7}$ .

3.  $2x + 8 = x - 4$   
 $x + 8 = -4$   
 $x = -12$

4.  $5x + 2 = 8 - 3x$   
 $8x + 2 = 8$   
 $8x = 6$   
 $x = \frac{3}{4}$

5.  $3r + 2 - 5(r + 1) = 6r + 4$   
 $3r + 2 - 5r - 5 = 6r + 4$   
 $-3 - 2r = 6r + 4$   
 $-3 = 8r + 4$   
 $-7 = 8r$   
 $-\frac{7}{8} = r$

The solution is  $-\frac{7}{8}$ .

6.  $5(a + 3) + 4a - 5 = -(2a - 4)$   
 $5a + 15 + 4a - 5 = -2a + 4$   
 $9a + 10 = -2a + 4$   
 $11a + 10 = 4$   
 $11a = -6$   
 $a = -\frac{6}{11}$

7.  $2[m - (4 + 2m) + 3] = 2m + 2$   
 $2[m - 4 - 2m + 3] = 2m + 2$   
 $2[-m - 1] = 2m + 2$   
 $-2m - 2 = 2m + 2$   
 $-2m = 2m + 4$   
 $-4m = 4$   
 $m = -1$

The solution is -1.

8.  $4[2p - (3 - p) + 5] = -7p - 2$   
 $4[2p - 3 + p + 5] = -7p - 2$   
 $4[3p + 2] = -7p - 2$   
 $12p + 8 = -7p - 2$   
 $19p + 8 = -2$   
 $19p = -10$   
 $p = -\frac{10}{19}$

9.  $x^2 + 5x + 6 = 0$   
 $(x + 3)(x + 2) = 0$   
 $x + 3 = 0$  or  $x + 2 = 0$   
 $x = -3$  or  $x = -2$

The solutions are -3 and -2.

10.  $x^2 = 3 + 2x$   
 $x^2 - 2x - 3 = 0$   
 $(x - 3)(x + 1) = 0$   
 $x - 3 = 0$  or  $x + 1 = 0$   
 $x = 3$  or  $x = -1$

The solutions are 3 and -1.

11.  $m^2 + 16 = 8m$   
 $m^2 - 8m + 16 = 0$   
 $(m)^2 - 2(4m) + (4)^2 = 0$   
 $(m - 4)^2 = 0$   
 $m - 4 = 0$   
 $m = 4$

The solution is 4.

12.  $2k^2 - k = 10$   
 $2k^2 - k - 10 = 0$   
 $(2k - 5)(k + 2) = 0$   
 $2k - 5 = 0$  or  $k + 2 = 0$   
 $k = \frac{5}{2}$  or  $k = -2$

The solutions are  $\frac{5}{2}$  and -2.

$$\begin{aligned}
 13. \quad & 6x^2 - 5x = 4 \\
 & 6x^2 - 5x - 4 = 0 \\
 & (3x - 4)(2x + 1) = 0 \\
 & 3x - 4 = 0 \quad \text{or} \quad 2x + 1 = 0 \\
 & 3x = 4 \qquad \qquad 2x = -1 \\
 & x = \frac{4}{3} \quad \text{or} \quad x = -\frac{1}{2}
 \end{aligned}$$

The solutions are  $\frac{4}{3}$  and  $-\frac{1}{2}$ .

$$\begin{aligned}
 14. \quad & m(m - 7) = -10 \\
 & m^2 - 7m + 10 = 0 \\
 & (m - 5)(m - 2) = 0 \\
 & m - 5 = 0 \quad \text{or} \quad m - 2 = 0 \\
 & m = 5 \quad \text{or} \quad m = 2
 \end{aligned}$$

The solutions are 5 and 2.

$$\begin{aligned}
 15. \quad & 9x^2 - 16 = 0 \\
 & (3x)^2 - (4)^2 = 0 \\
 & (3x + 4)(3x - 4) = 0 \\
 & 3x + 4 = 0 \quad \text{or} \quad 3x - 4 = 0 \\
 & 3x = -4 \qquad \qquad 3x = 4 \\
 & x = -\frac{4}{3} \quad \text{or} \quad x = \frac{4}{3}
 \end{aligned}$$

The solutions are  $-\frac{4}{3}$  and  $\frac{4}{3}$ .

$$\begin{aligned}
 16. \quad & z(2z + 7) = 4 \\
 & 2z^2 + 7z - 4 = 0 \\
 & (2z - 1)(z + 4) = 0 \\
 & 2z - 1 = 0 \quad \text{or} \quad z + 4 = 0 \\
 & z = \frac{1}{2} \quad \text{or} \quad z = -4
 \end{aligned}$$

The solutions are  $\frac{1}{2}$  and  $-4$ .

$$\begin{aligned}
 17. \quad & 12y^2 - 48y = 0 \\
 & 12y(y) - 12y(4) = 0 \\
 & 12y(y - 4) = 0 \\
 & 12y = 0 \quad \text{or} \quad y - 4 = 0 \\
 & y = 0 \quad \text{or} \quad y = 4
 \end{aligned}$$

The solutions are 0 and 4.

$$18. \quad 3x^2 - 5x + 1 = 0$$

Use the quadratic formula.

$$\begin{aligned}
 x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)} \\
 &= \frac{5 \pm \sqrt{25 - 12}}{6} \\
 x &= \frac{5 + \sqrt{13}}{6} \quad \text{or} \quad x = \frac{5 - \sqrt{13}}{6} \\
 &\approx 1.434 \qquad \qquad \approx .232
 \end{aligned}$$

The solutions are  $\frac{5 + \sqrt{13}}{6} \approx 1.434$  and

$$\frac{5 - \sqrt{13}}{6} \approx .232.$$

$$\begin{aligned}
 19. \quad & 2m^2 = m + 4 \\
 & 2m^2 - m - 4 = 0
 \end{aligned}$$

Use the quadratic formula.

$$\begin{aligned}
 x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-4)}}{2(2)} \\
 x &= \frac{1 \pm \sqrt{1 + 32}}{4} \\
 x &= \frac{1 \pm \sqrt{33}}{4}
 \end{aligned}$$

The solutions are  $\frac{1 + \sqrt{33}}{4} \approx 1.686$  and

$$\frac{1 - \sqrt{33}}{4} \approx -1.186.$$

$$20. \quad p^2 + p - 1 = 0$$

$$\begin{aligned}
 p &= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} \\
 &= \frac{-1 \pm \sqrt{5}}{2}
 \end{aligned}$$

The solutions are  $\frac{-1 + \sqrt{5}}{2} \approx .618$  and

$$\frac{-1 - \sqrt{5}}{2} \approx -1.618.$$

$$\begin{aligned}
 21. \quad & k^2 - 10k = 20 \\
 & k^2 - 10k + 20 = 0
 \end{aligned}$$

$$k = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(20)}}{2(1)}$$

$$k = \frac{10 \pm \sqrt{100 - 80}}{2}$$

$$k = \frac{10 \pm \sqrt{20}}{2}$$

$$k = \frac{10 \pm \sqrt{4}\sqrt{5}}{2}$$

$$k = \frac{10 \pm 2\sqrt{5}}{2}$$

$$k = \frac{2(5 \pm 2\sqrt{5})}{2}$$

$$k = 5 \pm \sqrt{5}$$

The solutions are  $5 + \sqrt{5} \approx 7.236$  and

$$5 - \sqrt{5} \approx 2.764.$$

$$22. 2x^2 + 12x + 5 = 0$$

$$\begin{aligned} x &= \frac{-12 \pm \sqrt{(12)^2 - 4(2)(5)}}{2(2)} \\ &= \frac{-12 \pm \sqrt{104}}{4} = \frac{-12 \pm \sqrt{4 \cdot 26}}{4} \\ &= \frac{-12 \pm \sqrt{4}\sqrt{26}}{4} = \frac{-12 \pm 2\sqrt{26}}{4} \\ &= \frac{2(-6 \pm \sqrt{26})}{2 \cdot 2} = \frac{-6 \pm \sqrt{26}}{2} \end{aligned}$$

The solutions are  $\frac{-6 + \sqrt{26}}{2} \approx -0.450$  and

$$\frac{-6 - \sqrt{26}}{2} \approx -5.550.$$

$$23. 2r^2 - 7r + 5 = 0$$

$$(2r - 5)(r - 1) = 0$$

$$2r - 5 = 0 \quad \text{or} \quad r - 1 = 0$$

$$2r = 5$$

$$r = \frac{5}{2} \quad \text{or} \quad r = 1$$

The solutions are  $\frac{5}{2}$  and 1.

$$24. 2x^2 - 7x + 30 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(30)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{49 - 240}}{4}$$

$$x = \frac{7 \pm \sqrt{-191}}{4}$$

Since there is a negative number under the radical sign,  $\sqrt{-191}$  is not a real number. Thus, there are no real-number solutions.

$$25. 3k^2 + k = 6$$

$$3k^2 + k - 6 = 0$$

$$k = \frac{-1 \pm \sqrt{1 - 4(3)(-6)}}{2(3)}$$

$$= \frac{-1 \pm \sqrt{73}}{6}$$

The solutions are  $\frac{-1 + \sqrt{73}}{6} \approx 1.257$  and

$$\frac{-1 - \sqrt{73}}{6} \approx -1.591.$$

$$26. 5m^2 + 5m = 0$$

$$5m(m + 1) = 0$$

$$5m = 0 \quad \text{or} \quad m + 1 = 0$$

$$m = 0 \quad \text{or} \quad m = -1$$

The solutions are 0 and -1.

$$27. \frac{3x - 2}{7} = \frac{x + 2}{5}$$

$$35 \left( \frac{3x - 2}{7} \right) = 35 \left( \frac{x + 2}{5} \right)$$

$$5(3x - 2) = 7(x + 2)$$

$$15x - 10 = 7x + 14$$

$$8x = 24$$

$$x = 3$$

$$28. \frac{x}{3} - 7 = 6 - \frac{3x}{4}$$

Multiply both sides by 12, the least common denominator of 3 and 4.

$$12 \left( \frac{x}{3} - 7 \right) = 12 \left( 6 - \frac{3x}{4} \right)$$

$$12 \left( \frac{x}{3} \right) - (12)(7) = (12)(6) - (12) \left( \frac{3x}{4} \right)$$

$$4x - 84 = 72 - 9x$$

$$13x - 84 = 72$$

$$13x = 156$$

$$x = 12$$

The solution is 12.

$$29. \frac{4}{x - 3} - \frac{8}{2x + 5} + \frac{3}{x - 3} = 0$$

$$\frac{4}{x - 3} + \frac{3}{x - 3} - \frac{8}{2x + 5} = 0$$

$$\frac{7}{x - 3} - \frac{8}{2x + 5} = 0$$

Multiply both sides by  $(x - 3)(2x + 5)$ . Note that  $x \neq 3$  and  $x \neq -\frac{5}{2}$ .

$$(x - 3)(2x + 5) \left( \frac{7}{x - 3} - \frac{8}{2x + 5} \right) = (x - 3)(2x + 5)(0)$$

$$7(2x + 5) - 8(x - 3) = 0$$

$$14x + 35 - 8x + 24 = 0$$

$$6x + 59 = 0$$

$$6x = -59$$

$$x = -\frac{59}{6}$$

Note: It is especially important to check solutions of equations that involve rational expressions. Here, a check shows that  $-\frac{59}{6}$  is a solution.