In Exercises 55 – 59, x and y are in quadrant I, so we know that $\sec x = \frac{3}{2} \quad \Rightarrow \quad \cos x = \frac{2}{3}$, so $\sin x = \frac{\sqrt{5}}{3}$ and $\tan x = \frac{\sqrt{5}}{2}$. Also, $\csc y = 3 \quad \Rightarrow \quad \sin y = \frac{1}{3}$, and so $\cos y = \frac{2\sqrt{2}}{3}$, and $\tan y = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$.

55. $\sin(x+y) = \sin x \cos y + \cos x \sin y = \frac{\sqrt{5}}{3} \cdot \frac{2\sqrt{2}}{3} + \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}(1 + \sqrt{10})$.

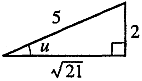
57. $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{\sqrt{5}}{2} + \frac{\sqrt{2}}{4}}{1 - \left(\frac{\sqrt{5}}{2}\right)\left(\frac{\sqrt{2}}{4}\right)} = \frac{\frac{\sqrt{5}}{2} + \frac{\sqrt{2}}{4}}{1 - \left(\frac{\sqrt{5}}{2}\right)\left(\frac{\sqrt{2}}{4}\right)} \cdot \frac{8}{8}$$
$$= \frac{2(2\sqrt{5}+\sqrt{2})}{8-\sqrt{10}} \cdot \frac{8+\sqrt{10}}{8+\sqrt{10}} = \frac{2}{3}\left(\sqrt{2} + \sqrt{5}\right).$$

59. $\cos \frac{y}{2} = \sqrt{\frac{1+\cos y}{2}} = \sqrt{\frac{1 + \left(\frac{2\sqrt{2}}{3}\right)}{2}} = \sqrt{\frac{3+2\sqrt{2}}{6}}$ (since cosine is positive in quadrant I).

61. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$.

63. $\cos(\tan^{-1}\sqrt{3}) = \cos \frac{\pi}{3} = \frac{1}{2}$.

65. Let $u = \sin^{-1}\frac{2}{5}$ and so $\sin u = \frac{2}{5}$. Then from the triangle $\tan(\sin^{-1}\frac{2}{5}) = \tan u = \frac{2}{\sqrt{21}}$.



67. $\cos(2\sin^{-1}\frac{1}{3}) = 1 - 2\sin^2(\sin^{-1}\frac{1}{3}) = 1 - 2\left(\frac{1}{3}\right)^2 = \frac{7}{9}$.