2. $\dfrac{1}{1-\sin^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x$

4. $\dfrac{1+\sec x}{\sec x} = \dfrac{1}{\sec x} + 1 = 1 + \cos x = (1+\cos x) \cdot \dfrac{1-\cos x}{1-\cos x} = \dfrac{1-\cos^2 x}{1-\cos x} = \dfrac{\sin^2 x}{1-\cos x}$

6. $(1-\tan x)(1-\cot x) = 1 - \cot x - \tan x + \tan x \cot x = 2 - (\cot x + \tan x)$

   $= 2 - \left(\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}\right) = 2 - \dfrac{\cos^2 x + \sin^2 x}{\cos x \sin x} = 2 - \dfrac{1}{\cos x \sin x} = 2 - \sec x \csc x$

8. $(\tan x + \cot x)^2 = \left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)^2 = \left(\dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)^2 = \left(\dfrac{1}{\cos x \sin x}\right)^2$

   $= (\sec x \csc x)^2 = \csc^2 x \sec^2 x$

10. $\dfrac{\cos(x+y)}{\cos x \sin y} = \dfrac{\cos x \cos y - \sin x \sin y}{\cos x \sin y} = \dfrac{\cos x \cos y}{\cos x \sin y} - \dfrac{\sin x \sin y}{\cos x \sin y} = \dfrac{\cos y}{\sin y} - \dfrac{\sin x}{\cos x} = \cot y - \tan x$

12. $\dfrac{\sin(x+y) + \sin(x-y)}{\cos(x+y) + \cos(x-y)} = \dfrac{2\sin\left(\frac{(x+y)+(x-y)}{2}\right)\cos\left(\frac{(x+y)-(x-y)}{2}\right)}{2\cos\left(\frac{(x+y)+(x-y)}{2}\right)\cos\left(\frac{(x+y)-(x-y)}{2}\right)} = \dfrac{2\sin x \cos y}{2\cos x \cos y} = \dfrac{\sin x}{\cos x} = \tan x$