Discovery Project

EXPONENTIAL EXPLOSION

To see how difficult it is to comprehend exponential growth, let's try a thought experiment.

Suppose you put a penny in your piggy bank today, two pennies tomorrow, four pennies the next day, and so on, doubling the number of pennies you add to the bank each day (see the table). How many pennies will you put in your piggy bank on day 30? The answer is 2^{30} pennies. That's simple, but can you guess how many dollars that is? 2^{30} pennies is more than 10 million dollars!

Day	Pennies
0	1
1	2
2	4
3	8
4	16
.	.
.	.
.	.
n	2^n
.	.
.	.
.	.

As you can see, the exponential function $f(x) = 2^x$ grows extremely fast. This is the principle behind atomic explosions. An atom splits releasing two neutrons which cause two atoms to split, each releasing two neutrons, causing four atoms to split, and so on. At the nth stage 2^n atoms split—an exponential explosion!

As we know, populations grow exponentially. Let's see what this means for a type of bacteria that splits every minute. Suppose that at 12:00 noon a single bacterium colonizes a discarded food can. The bacterium and his descendants are all happy, but they fear the time when the can is completely full of bacteria—doomsday.

1. How many bacteria are in the can at 12:05? at 12:10?
2. The can is completely full of bacteria at 1:00 P.M. At what time was the can only half full of bacteria?
3. When the can is exactly half full, the president of the bacteria colony reassures his constituents that doomsday is far away—after all, there is as much room left in the can as has been used in the entire previous history of the colony. Is the president correct? How much time is left before doomsday?
4. When the can is one-quarter full, how much time remains till doomsday?
5. A wise bacterium decides to start a new colony in another can and slow down splitting time to 2 minutes. How much time does this new colony have?