

63. Completing the square gives  $x^2 + y^2 - 4x + 10y + 13 = 0 \Leftrightarrow$   
 $x^2 - 4x + \_\_ + y^2 + 10y + \_\_ = -13 \Leftrightarrow$   
 $x^2 - 4x + \left(\frac{-4}{2}\right)^2 + y^2 + 10y + \left(\frac{10}{2}\right)^2 = -13 + \left(\frac{4}{2}\right)^2 + \left(\frac{10}{2}\right)^2 \Leftrightarrow$   
 $x^2 - 4x + 4 + y^2 + 10y + 25 = -13 + 4 + 25 \Leftrightarrow (x-2)^2 + (y+5)^2 = 16.$

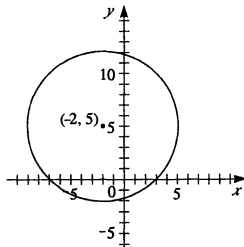
Thus, the center is  $(2, -5)$ , and the radius is 4.

65. Completing the square gives  $x^2 + y^2 + x = 0 \Leftrightarrow x^2 + x + \_\_ + y^2 = 0 \Leftrightarrow$   
 $x^2 + x + \left(\frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2 \Leftrightarrow x^2 + x + \frac{1}{4} + y^2 = \frac{1}{4} \Leftrightarrow \left(x + \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}.$  Thus, the  
center:  $\left(-\frac{1}{2}, 0\right)$ , and the radius is  $\frac{1}{2}$ .

67. Completing the square gives  $x^2 + y^2 - \frac{1}{2}x + \frac{1}{2}y = \frac{1}{8} \Leftrightarrow x^2 - \frac{1}{2}x + \_\_ + y^2 + \frac{1}{2}y + \_\_ = \frac{1}{8}$   
 $\Leftrightarrow x^2 - \frac{1}{2}x + \left(\frac{-1/2}{2}\right)^2 + y^2 + \frac{1}{2}y + \left(\frac{1/2}{2}\right)^2 = \frac{1}{8} + \left(\frac{-1/2}{2}\right)^2 + \left(\frac{1/2}{2}\right)^2 \Leftrightarrow$   
 $x^2 - \frac{1}{2}x + \frac{1}{16} + y^2 + \frac{1}{2}y + \frac{1}{16} = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = \frac{2}{8} = \frac{1}{4} \Leftrightarrow \left(x - \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{1}{4}.$  Thus, the  
center is  $\left(\frac{1}{4}, -\frac{1}{4}\right)$ , and the radius is  $\frac{1}{2}$ .

69. Completing the square gives  $x^2 + y^2 + 4x - 10y = 21 \Leftrightarrow$   
 $x^2 + 4x + \_\_ + y^2 - 10y + \_\_ = 21 \Leftrightarrow$   
 $x^2 + 4x + \left(\frac{4}{2}\right)^2 + y^2 - 10y + \left(\frac{-10}{2}\right)^2 = 21 + \left(\frac{4}{2}\right)^2 + \left(\frac{-10}{2}\right)^2 \Leftrightarrow$   
 $(x+2)^2 + (y-5)^2 = 21 + 4 + 25 = 50.$

Thus, the center is  $(-2, 5)$ , and the radius is  $\sqrt{50} = 5\sqrt{2}$ .



71. Completing the square gives  $x^2 + y^2 + 6x - 12y + 45 = 0 \Leftrightarrow$   
 $x^2 + 6x + \_\_ + y^2 - 12y + \_\_ = -45 \Leftrightarrow$   
 $x^2 + 6x + \left(\frac{6}{2}\right)^2 + y^2 - 12y + \left(\frac{-12}{2}\right)^2 = -45 + \left(\frac{6}{2}\right)^2 + \left(\frac{-12}{2}\right)^2 \Leftrightarrow$   
 $(x+3)^2 + (y-6)^2 = -45 + 9 + 36 = 0.$  Thus, the center is  $(-3, 6)$ ,  
and the radius is 0. This is a degenerate circle whose graph consists only  
of the point  $(-3, 6)$ .

