

77. Completing the square gives $x^2 + y^2 - 4y - 12 = 0 \Leftrightarrow x^2 + y^2 - 4y + \underline{\quad} = 12 \Leftrightarrow x^2 + y^2 - 4y + \left(\frac{-4}{2}\right)^2 = 12 + \left(\frac{-4}{2}\right)^2 \Leftrightarrow x^2 + (y - 2)^2 = 16$. Thus, the center is $(0, 2)$, and the radius is 4. So the circle $x^2 + y^2 = 4$, with center $(0, 0)$ and radius 2, sits completely inside the larger circle. Thus, the area is $\pi 4^2 - \pi 2^2 = 16\pi - 4\pi = 12\pi$.

79. Completing the square gives $x^2 + y^2 + ax + by + c = 0 \Leftrightarrow x^2 + ax + \underline{\quad} + y^2 + by + \underline{\quad} = -c \Leftrightarrow x^2 + ax + \left(\frac{a}{2}\right)^2 + y^2 + by + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 \Leftrightarrow \left(x + \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 = -c + \frac{a^2 + b^2}{4}$.

This equation represents a circle only when $-c + \frac{a^2 + b^2}{4} > 0$. This equation represents a point when $-c + \frac{a^2 + b^2}{4} = 0$, and this equation represents the empty set when $-c + \frac{a^2 + b^2}{4} < 0$.

When the equation represents a circle, the center is $\left(-\frac{a}{2}, -\frac{b}{2}\right)$, and the radius is

$$\sqrt{-c + \frac{a^2 + b^2}{4}} = \frac{1}{2} \sqrt{a^2 + b^2 - 4ac}.$$