

80. (a) (i) $(x - 2)^2 + (y - 1)^2 = 9$, the center is at $(2, 1)$, and the radius is 3.
 $(x - 6)^2 + (y - 4)^2 = 16$, the center is at $(6, 4)$, and the radius is 4.

The distance between centers is

$$\sqrt{(2 - 6)^2 + (1 - 4)^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5.$$

Since $5 < 3 + 4$, these circles intersect.

- (ii) $x^2 + (y - 2)^2 = 4$, the center is at $(0, 2)$, and the radius is 2.
 $(x - 5)^2 + (y - 14)^2 = 9$, the center is at $(5, 14)$, and the radius is 3.

The distance between centers is

$$\sqrt{(0 - 5)^2 + (2 - 14)^2} = \sqrt{(-5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$$

Since $13 > 2 + 3$, these circles do not intersect.

- (iii) $(x - 3)^2 + (y + 1)^2 = 1$, the center is at $(3, -1)$, and the radius is 1.
 $(x - 2)^2 + (y - 2)^2 = 25$, the center is at $(2, 2)$, and the radius is 5.

The distance between centers is

$$\sqrt{(3 - 2)^2 + (-1 - 2)^2} = \sqrt{1^2 + (-3)^2} = \sqrt{1 + 9} = \sqrt{10}.$$

Since $\sqrt{10} < 1 + 5$, these circles intersect.

- (b) As shown in the diagram, if two circles intersect, then the centers of the circles and one point of intersection form a triangle. So because in any triangle each side has length less than the sum of the other two, the two circles will intersect only if distance between their centers, d , is less than or equal to the sum of the radii, r_1 and r_2 . That is, the circles will intersect if $d \leq r_1 + r_2$.

