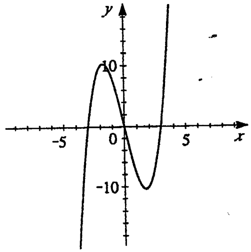
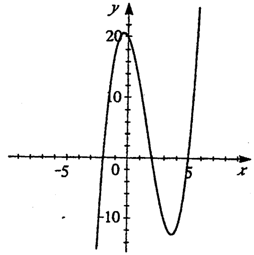
3. $P(x) = x^3 - 9x = x(x-3)(x+3)$



5. $P(x) = x^3 - 5x^2 - 4x + 20$
$= (x-5)(x-2)(x+2)$



11.

3	1	−1	1	−11
		3	6	21
	1	2	7	10

Therefore, $Q(x) = x^2 + 2x + 7$, and $R(x) = 10$.

13.

$$\begin{array}{r} x \quad -3 \qquad\qquad\quad \\ x^2+2x-5 \overline{\big) x^3 \quad - x^2 - 11x \quad + 6} \\ \underline{x^3 + 2x^2 \quad - 5x \qquad} \\ -3x^2 \quad - 6x \quad + 6 \\ \underline{-3x^2 \quad - 6x + 15} \\ -9 \end{array}$$

Therefore, $Q(x) = x - 3$, and $R(x) = -9$.

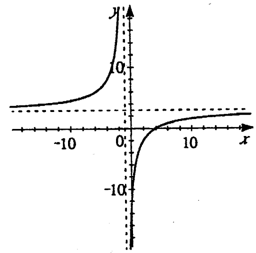
15.

−5	1	0	−25	4	15
		−5	25	0	−20
	1	−5	0	4	−5

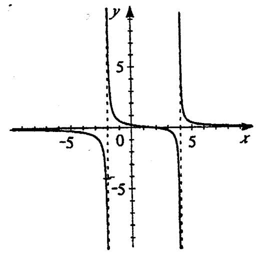
Therefore, $Q(x) = x^3 - 5x^2 + 4$, and $R(x) = -5$.

27. Since the zeros are $-\frac{1}{2}$, 2, and 3, a factorization is $P(x) = C\left(x + \frac{1}{2}\right)(x-2)(x-3)$
$= \frac{C}{2}(2x+1)(x^2 - 5x + 6) = \frac{C}{2}(2x^3 - 10x^2 + 12x + x^2 - 5x + 6) = \frac{C}{2}(2x^3 - 9x^2 + 7x + 6)$.
Since the constant coefficient is 12, $\frac{C}{2}(6) = 12 \quad \Leftrightarrow \quad C = 4$, and so the polynomial is
$P(x) = 4x^3 - 18x^2 + 14x + 12$.

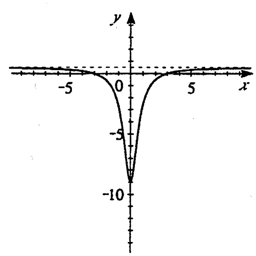
45. $r(x) = \dfrac{3x - 12}{x + 1}$. When $x = 0$, we have $r(0) = \frac{-12}{1} = -12$, so the y-intercept is -12. Since $y = 0$, when $3x - 12 = 0 \quad \Leftrightarrow \quad x = 4$, the x-intercept is 4. The vertical asymptote is $x = -1$. Because the degree of the denominator and numerator are the same, the horizontal asymptote is $y = \frac{3}{1} = 3$.



47. $r(x) = \dfrac{x - 2}{x^2 - 2x - 8} = \dfrac{x - 2}{(x + 2)(x - 4)}$. When $x = 0$, we have $r(0) = \frac{-2}{-8} = \frac{1}{4}$, so the y-intercept is $\frac{1}{4}$. When $y = 0$, we have $x - 2 = 0 \quad \Leftrightarrow \quad x = 2$, so the x-intercepts is 2. The vertical asymptotes occur when $x = -2$ and $x = 4$. The horizontal than the degree of the numerator.



49. $r(x) = \dfrac{x^2 - 9}{2x^2 + 1} = \dfrac{(x + 3)(x - 3)}{2x^2 + 1}$. When $x = 0$, we have $r(0) = \frac{-9}{1}$, so the y-intercept is -9. When $y = 0$, we have $x^2 - 9 = 0 \quad \Leftrightarrow \quad x = \pm 3$ so the x-intercepts are -3 and 3. Since $2x^2 + 1 > 0$, the denominator is never zero so there are no vertical asymptotes. The horizontal asymptote is at $y = \frac{1}{2}$ because the degree of the denominator and numerator are the same.



55. The graphs $y = x^4 + x^2 + 24x$ and $y = 6x^3 + 20$ intersect when $x^4 + x^2 + 24x = 6x^3 + 20 \quad \Leftrightarrow$ $x^4 - 6x^3 + x^2 + 24x - 20 = 0$. The possible rational zeros are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$.

1	1	−6	1	24	−20
		1	−5	−4	20
	1	−5	−4	20	0

$\Rightarrow \quad x = 1$ is a zero.

So $x^4 - 6x^3 + x^2 + 24x - 20 = (x - 1)(x^3 - 5x^2 - 4x + 20) = 0$. Continuing with the quotient:

1	1	−5	−4	20
		1	−4	−8
	1	−4	−8	12

2	1	−5	−4	20
		2	−6	−20
	1	−3	−10	0

$\Rightarrow \quad x = 2$ is a zero.

So $x^4 - 6x^3 + x^2 + 24x - 20 = (x - 1)(x - 2)(x^2 - 3x - 10)$
$= (x - 1)(x - 2)(x - 5)(x + 2) = 0$. Hence, the points of intersection are $(1, 26)$, $(2, 68)$, $(5, 770)$, and $(-2, -28)$.