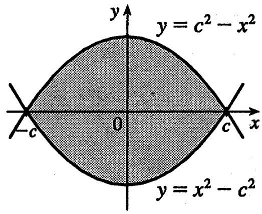
**35.** We first assume that $c > 0$, since $c$ can be replaced by $-c$ in both equations without changing the graphs, and if $c = 0$ the curves do not enclose a region. We see from the graph that the enclosed area $A$ lies between $x = -c$ and $x = c$, and by symmetry, it is equal to four times the area in the first quadrant.

The enclosed area is

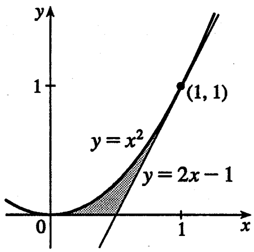



$$A = 4\int_0^c (c^2 - x^2)\,dx = 4\left[c^2x - \tfrac{1}{3}x^3\right]_0^c$$
$$= 4\left(c^3 - \tfrac{1}{3}c^3\right) = 4\left(\tfrac{2}{3}c^3\right) = \tfrac{8}{3}c^3$$

So $A = 576 \quad\Leftrightarrow\quad \frac{8}{3}c^3 = 576 \quad\Leftrightarrow\quad c^3 = 216 \quad\Leftrightarrow\quad c = \sqrt[3]{216} = 6.$

Note that $c = -6$ is another solution, since the graphs are the same.

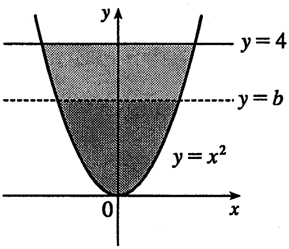
**36.**




We start by finding the equation of the tangent line to $y = x^2$ at the point $(1, 1)$: $y' = 2x$, so the slope of the tangent is $2(1) = 2$, and its equation is $y - 1 = 2(x - 1)$, or $y = 2x - 1$. We would need two integrals to integrate with respect to $x$, but only one to integrate with respect to $y$.

$$A = \int_0^1 \left[\tfrac{1}{2}(y+1) - \sqrt{y}\right] dy = \left[\tfrac{1}{4}y^2 + \tfrac{1}{2}y - \tfrac{2}{3}y^{3/2}\right]_0^1$$
$$= \tfrac{1}{4} + \tfrac{1}{2} - \tfrac{2}{3} = \tfrac{1}{12}$$

**37.**




By the symmetry of the problem, we consider only the first quadrant, where $y = x^2 \quad\Rightarrow\quad x = \sqrt{y}$. We are looking for a number $b$ such that $\int_0^b \sqrt{y}\,dy = \int_b^4 \sqrt{y}\,dy \quad\Rightarrow\quad \frac{2}{3}\left[y^{3/2}\right]_0^b = \frac{2}{3}\left[y^{3/2}\right]_b^4 \quad\Rightarrow$
$b^{3/2} = 4^{3/2} - b^{3/2} \quad\Rightarrow\quad 2b^{3/2} = 8 \quad\Rightarrow\quad b^{3/2} = 4 \quad\Rightarrow$
$b = 4^{2/3} \approx 2.52.$

**38.** (a) We want to choose $a$ so that $\displaystyle\int_1^a \frac{1}{x^2}\,dx = \int_a^4 \frac{1}{x^2}\,dx \quad\Rightarrow\quad \left[\frac{-1}{x}\right]_1^a = \left[\frac{-1}{x}\right]_a^4 \quad\Rightarrow\quad -\frac{1}{a} + 1 = -\frac{1}{4} + \frac{1}{a}$
$\displaystyle\Rightarrow\quad \frac{5}{4} = \frac{2}{a} \quad\Rightarrow\quad a = \frac{8}{5}.$

(b) The area under the curve $y = 1/x^2$ from $x = 1$ to $x = 4$ is $\frac{3}{4}$ [take $a = 4$ in the first integral in part (a)]. Now the line $y = b$ must intersect the curve $x = 1/\sqrt{y}$ and not the line $x = 4$, since the area under the line $y = 1/4^2$ from $x = 1$ to $x = 4$ is only $\frac{3}{16}$, which is less than half of $\frac{3}{4}$. We want to choose $b$ so that the upper area in the diagram is half of the total area under the curve $y = \dfrac{1}{x^2}$ from $x = 1$ to $x = 4$. This implies that

$\int_b^1 (1/\sqrt{y} - 1)\, dy = \frac{1}{2} \cdot \frac{3}{4} \quad\Rightarrow\quad \left[2\sqrt{y} - y\right]_b^1 = \frac{3}{8} \quad\Rightarrow$

$1 - 2\sqrt{b} + b = \frac{3}{8} \quad\Rightarrow\quad b - 2\sqrt{b} + \frac{5}{8} = 0$. Letting $c = \sqrt{b}$, we get

$c^2 - 2c + \frac{5}{8} = 0 \quad\Rightarrow\quad 8c^2 - 16c + 5 = 0$. Thus,

$c = \frac{16 \pm \sqrt{256 - 160}}{16} = 1 \pm \frac{\sqrt{6}}{4}$. But $c = \sqrt{b} < 1 \quad\Rightarrow\quad c = 1 - \frac{\sqrt{6}}{4} \quad\Rightarrow$

$b = c^2 = 1 + \frac{3}{8} - \frac{\sqrt{6}}{2} = \frac{1}{8}\left(11 - 4\sqrt{6}\right) \approx 0.1503$.