Tuesday, FEBRUARY 10, 2009

10th Annual American Mathematics Contest 10

AMC 10
CONTEST A

THE MATHEMATICAL ASSOCIATION OF AMERICA
American Mathematics Competitions

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR GIVES THE SIGNAL TO BEGIN.
2. This is a 25-question, multiple choice test. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Mark your answer to each problem on the AMC 10 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, ruler, compass, protractor, and erasers. No calculators are allowed. No problems on the test will *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record certain information on the answer form. When your proctor gives the signal, begin working the problems. You will have **75 MINUTES** to complete the test.
8. When you finish the exam, *sign your name* in the space provided on the Answer Form.

Students who score 120 or above or finish in the top 1% on this AMC 10 will be invited to take the 27th annual American Invitational Mathematics Examination (AIME) on Tuesday, March 17, 2009 or Wednesday, April 1, 2009. More details about the AIME and other information are on the back page of this test booklet.

The Committee on the American Mathematics Competitions (CAMC) reserves the right to re-examine students before deciding whether to grant official status to their scores. The CAMC also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

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1. One can holds 12 ounces of soda. What is the minimum number of cans needed to provide a gallon (128 ounces) of soda?

   (A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

2. Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes, and quarters. Which of the following could *not* be the total value of the four coins, in cents?

   (A) 15 **(B)** 25 **(C)** 35 **(D)** 45 **(E)** 55

3. Which of the following is equal to $1 + \dfrac{1}{1 + \frac{1}{1+1}}$?

   (A) $\dfrac{5}{4}$ **(B)** $\dfrac{3}{2}$ **(C)** $\dfrac{5}{3}$ **(D)** 2 **(E)** 3

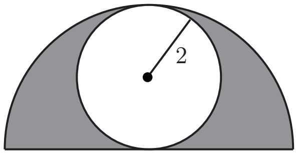
4. Eric plans to compete in a triathlon. He can average 2 miles per hour in the $\frac{1}{4}$-mile swim and 6 miles per hour in the 3-mile run. His goal is to finish the triathlon in 2 hours. To accomplish his goal what must his average speed, in miles per hour, be for the 15-mile bicycle ride?

   (A) $\dfrac{120}{11}$ **(B)** 11 **(C)** $\dfrac{56}{5}$ **(D)** $\dfrac{45}{4}$ **(E)** 12

5. What is the sum of the digits of the square of 111,111,111 ?

   (A) 18 **(B)** 27 **(C)** 45 **(D)** 63 **(E)** 81

6. A circle of radius 2 is inscribed in a semicircle, as shown. The area inside the semicircle but outside the circle is shaded. What fraction of the semicircle's area is shaded?



   (A) $\dfrac{1}{2}$ **(B)** $\dfrac{\pi}{6}$ **(C)** $\dfrac{2}{\pi}$ **(D)** $\dfrac{2}{3}$ **(E)** $\dfrac{3}{\pi}$

7. A carton contains milk that is 2% fat, an amount that is 40% less fat than the amount contained in a carton of whole milk. What is the percentage of fat in whole milk?

(A) $\frac{12}{5}$ **(B)** 3 **(C)** $\frac{10}{3}$ **(D)** 38 **(E)** 42

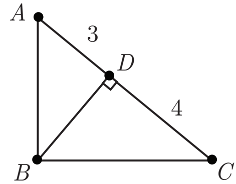
8. Three generations of the Wen family are going to the movies, two from each generation. The two members of the youngest generation receive a 50% discount as children. The two members of the oldest generation receive a 25% discount as senior citizens. The two members of the middle generation receive no discount. Grandfather Wen, whose senior ticket costs \$6.00, is paying for everyone. How many dollars must he pay?

(A) 34 **(B)** 36 **(C)** 42 **(D)** 46 **(E)** 48

9. Positive integers a, b, and 2009, with $a < b < 2009$, form a geometric sequence with an integer ratio. What is a?

(A) 7 **(B)** 41 **(C)** 49 **(D)** 287 **(E)** 2009

10. Triangle ABC has a right angle at B. Point D is the foot of the altitude from B, $AD = 3$, and $DC = 4$. What is the area of $\triangle ABC$?

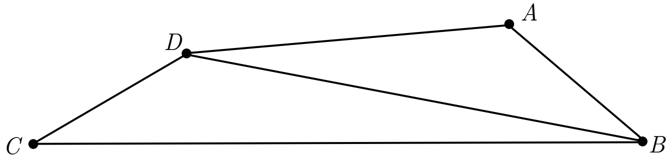


(A) $4\sqrt{3}$ **(B)** $7\sqrt{3}$ **(C)** 21 **(D)** $14\sqrt{3}$ **(E)** 42

11. One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

(A) 8 **(B)** 27 **(C)** 64 **(D)** 125 **(E)** 216

12. In quadrilateral $ABCD$, $AB = 5$, $BC = 17$, $CD = 5$, $DA = 9$, and BD is an integer. What is BD?

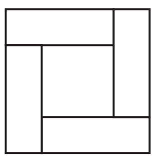


(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

13. Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n)?

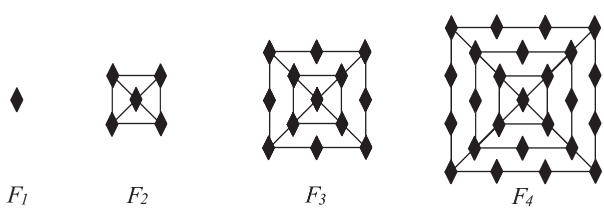
(A) P^2Q **(B)** P^nQ^m **(C)** P^nQ^{2m} **(D)** $P^{2m}Q^n$ **(E)** $P^{2n}Q^m$

14. Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?



(A) 3 **(B)** $\sqrt{10}$ **(C)** $2+\sqrt{2}$ **(D)** $2\sqrt{3}$ **(E)** 4

15. The figures F_1, F_2, F_3 and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



(A) 401 **(B)** 485 **(C)** 585 **(D)** 626 **(E)** 761

16. Let a, b, c, and d be real numbers with $|a-b|=2$, $|b-c|=3$, and $|c-d|=4$. What is the sum of all possible values of $|a-d|$?

(A) 9 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 24

17. Rectangle $ABCD$ has $AB=4$ and $BC=3$. Segment EF is constructed through B so that $\overline{EF}\perp\overline{DB}$, and A and C lie on $\overline{DE}$ and $\overline{DF}$, respectively. What is EF?

(A) 9 **(B)** 10 **(C)** $\frac{125}{12}$ **(D)** $\frac{103}{9}$ **(E)** 12

18. At Jefferson Summer Camp, 60% of the children play soccer, 30% of the children swim, and 40% of the soccer players swim. To the nearest whole percent, what percent of the non-swimmers play soccer?

(A) 30% **(B)** 40% **(C)** 49% **(D)** 51% **(E)** 70%

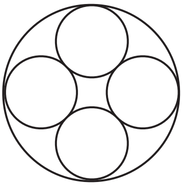
19. Circle A has radius 100. Circle B has an integer radius $r<100$ and remains internally tangent to circle A as it rolls once around the circumference of circle A. The two circles have the same points of tangency at the beginning and end of circle B's trip. How many possible values can r have?

(A) 4 **(B)** 8 **(C)** 9 **(D)** 50 **(E)** 90

20. Andrea and Lauren are 20 kilometers apart. They bike toward one another with Andrea traveling three times as fast as Lauren, and the distance between them decreasing at a rate of 1 kilometer per minute. After 5 minutes, Andrea stops biking because of a flat tire and waits for Lauren. After how many minutes from the time they started to bike does Lauren reach Andrea?

(A) 20 **(B)** 30 **(C)** 55 **(D)** 65 **(E)** 80

21. Many Gothic cathedrals have windows with portions containing a ring of congruent circles that are circumscribed by a larger circle. In the figure shown, the number of smaller circles is four. What is the ratio of the sum of the areas of the four smaller circles to the area of the larger circle?



(A) $3-2\sqrt{2}$ **(B)** $2-\sqrt{2}$ **(C)** $4(3-2\sqrt{2})$ **(D)** $\frac{1}{2}(3-\sqrt{2})$

(E) $2\sqrt{2}-2$

22. Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7?

(A) $\frac{1}{9}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{2}{11}$ **(E)** $\frac{1}{5}$

23. Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

(A) $\frac{9}{2}$ **(B)** $\frac{50}{11}$ **(C)** $\frac{21}{4}$ **(D)** $\frac{17}{3}$ **(E)** 6

24. Three distinct vertices of a cube are chosen at random. What is the probability that the plane determined by these three vertices contains points inside the cube?

(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{4}{7}$ **(D)** $\frac{5}{7}$ **(E)** $\frac{3}{4}$

25. For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10