71. (a) $f(x) = \log_2(\log_{10}x)$. Since the domain of $\log_2 x$ is the positive real numbers, we have: $\log_{10}x > 0 \quad \Leftrightarrow \quad x > 10^0 = 1$. Thus the domain of $f(x)$ is $(1, \infty)$.

(b) $y = \log_2(\log_{10}x) \quad \Leftrightarrow \quad 2^y = \log_{10}x \quad \Leftrightarrow \quad 10^{2^y} = x$. Thus $f^{-1}(x) = 10^{2^x}$.

73. (a) $f(x) = \dfrac{2^x}{1+2^x}$. $y = \dfrac{2^x}{1+2^x} \quad \Leftrightarrow \quad y + y\,2^x = 2^x \quad \Leftrightarrow \quad y = 2^x - y\,2^x = 2^x(1-y) \quad \Leftrightarrow$

$2^x = \dfrac{y}{1-y} \quad \Leftrightarrow \quad x = \log_2\left(\dfrac{y}{1-y}\right)$. Thus $f^{-1}(x) = \log_2\left(\dfrac{x}{1-x}\right)$.

(b) $\dfrac{x}{1-x} > 0$. Solving this using the methods from Chapter 2, we start with the endpoints of the potential intervals, 0 and 1.

Interval	$(-\infty, 0)$	$(0,1)$	$(1,\infty)$
Sign of x	$-$	$+$	$+$
Sign of $1-x$	$+$	$+$	$-$
Sign of $\dfrac{x}{1-x}$	$-$	$+$	$-$

Thus the domain of $f^{-1}(x)$ is $(0,1)$.

75. $\log(\log 10^{100}) = \log 100 = 2$

$\log(\log(\log 10^{\text{googol}})) = \log(\log(\text{googol})) = \log(\log 10^{100}) = \log(100) = 2$

77. The numbers between 1000 and 9999 (inclusive) each have 4 digits, while $\log 1000 = 3$ and $\log 10{,}000 = 4$. Since $[\![\log x]\!] = 3$ for all integers x where $1000 \le x < 10000$, the number of digits is $[\![\log x]\!] + 1$. Likewise, if x is an integer where $10^{n-1} \le x < 10^n$, then x would have n digits and $[\![\log x]\!] = n - 1$. Since $[\![\log x]\!] = n - 1 \quad \Leftrightarrow \quad n = [\![\log x]\!] + 1$, the number of digits in x is $[\![\log x]\!] + 1$.