R.4 Equations

1.
$$\begin{aligned} .2m - .5 &= .1m + .7 \\ 10(.2m - .5) &= 10(.1m + .7) \\ 2m - 5 &= m + 7 \\ m - 5 &= 7 \\ m &= 12 \end{aligned}$$

The solution is 12.

2. $\frac{5}{6}k - 2k + \frac{1}{3} = \frac{2}{3}$

Multiply both sides of the equation by 6.

$$\begin{aligned} 6\left(\frac{5}{6}k\right) - 6(2k) + 6\left(\frac{1}{3}\right) &= 6\left(\frac{2}{3}\right) \\ 5k - 12k + 2 &= 4 \\ -7k + 2 &= 4 \\ -7k &= 2 \\ k &= -\frac{2}{7} \end{aligned}$$

The solution is $-\frac{2}{7}$.

3.
$$\begin{aligned} 2x + 8 &= x - 4 \\ x + 8 &= -4 \\ x &= -12 \end{aligned}$$

4.
$$\begin{aligned} 5x + 2 &= 8 - 3x \\ 8x + 2 &= 8 \\ 8x &= 6 \\ x &= \frac{3}{4} \end{aligned}$$

5.
$$\begin{aligned} 3r + 2 - 5(r + 1) &= 6r + 4 \\ 3r + 2 - 5r - 5 &= 6r + 4 \\ -3 - 2r &= 6r + 4 \\ -3 &= 8r + 4 \\ -7 &= 8r \\ -\frac{7}{8} &= r \end{aligned}$$

The solution is $-\frac{7}{8}$.

6.
$$\begin{aligned} 5(a + 3) + 4a - 5 &= -(2a - 4) \\ 5a + 15 + 4a - 5 &= -2a + 4 \\ 9a + 10 &= -2a + 4 \\ 11a + 10 &= 4 \\ 11a &= -6 \\ a &= -\frac{6}{11} \end{aligned}$$

7.
$$\begin{aligned} 2[m - (4 + 2m) + 3] &= 2m + 2 \\ 2[m - 4 - 2m + 3] &= 2m + 2 \\ 2[-m - 1] &= 2m + 2 \\ -2m - 2 &= 2m + 2 \\ -2m &= 2m + 4 \\ -4m &= 4 \\ m &= -1 \end{aligned}$$

The solution is -1.

8.
$$\begin{aligned} 4[2p - (3 - p) + 5] &= -7p - 2 \\ 4[2p - 3 + p + 5] &= -7p - 2 \\ 4[3p + 2] &= -7p - 2 \\ 12p + 8 &= -7p - 2 \\ 19p + 8 &= -2 \\ 19p &= -10 \\ p &= -\frac{10}{19} \end{aligned}$$

9.
$$\begin{aligned} x^2 + 5x + 6 &= 0 \\ (x + 3)(x + 2) &= 0 \\ x + 3 = 0 \quad &\text{or} \quad x + 2 = 0 \\ x = -3 \quad &\text{or} \quad x = -2 \end{aligned}$$

The solutions are -3 and -2.

10.
$$\begin{aligned} x^2 &= 3 + 2x \\ x^2 - 2x - 3 &= 0 \\ (x - 3)(x + 1) &= 0 \\ x - 3 = 0 \quad &\text{or} \quad x + 1 = 0 \\ x = 3 \quad &\text{or} \quad x = -1 \end{aligned}$$

The solutions are 3 and -1.

11.
$$\begin{aligned} m^2 + 16 &= 8m \\ m^2 - 8m + 16 &= 0 \\ (m)^2 - 2(4m) + (4)^2 &= 0 \\ (m - 4)^2 &= 0 \\ m - 4 &= 0 \\ m &= 4 \end{aligned}$$

The solution is 4.

12.
$$\begin{aligned} 2k^2 - k &= 10 \\ 2k^2 - k - 10 &= 0 \\ (2k - 5)(k + 2) &= 0 \\ 2k - 5 = 0 \quad &\text{or} \quad k + 2 = 0 \\ k = \frac{5}{2} \quad &\text{or} \quad k = -2 \end{aligned}$$

The solutions are $\frac{5}{2}$ and -2.

13.
$$6x^2 - 5x = 4$$
$$6x^2 - 5x - 4 = 0$$
$$(3x-4)(2x+1) = 0$$
$$3x - 4 = 0 \quad \text{or} \quad 2x + 1 = 0$$
$$3x = 4 \qquad 2x = -1$$
$$x = \frac{4}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

The solutions are $\frac{4}{3}$ and $-\frac{1}{2}$.

14.
$$m(m-7) = -10$$
$$m^2 - 7m + 10 = 0$$
$$(m-5)(m-2) = 0$$
$$m - 5 = 0 \quad \text{or} \quad m - 2 = 0$$
$$m = 5 \quad \text{or} \quad m = 2$$

The solutions are 5 and 2.

15.
$$9x^2 - 16 = 0$$
$$(3x)^2 - (4)^2 = 0$$
$$(3x+4)(3x-4) = 0$$
$$3x + 4 = 0 \quad \text{or} \quad 3x - 4 = 0$$
$$3x = -4 \qquad 3x = 4$$
$$x = -\frac{4}{3} \quad \text{or} \quad x = \frac{4}{3}$$

The solutions are $-\frac{4}{3}$ and $\frac{4}{3}$.

16.
$$z(2z+7) = 4$$
$$2z^2 + 7z - 4 = 0$$
$$(2z-1)(z+4) = 0$$
$$2z - 1 = 0 \quad \text{or} \quad z + 4 = 0$$
$$z = \frac{1}{2} \quad \text{or} \quad z = -4$$

The solutions are $\frac{1}{2}$ and -4.

17.
$$12y^2 - 48y = 0$$
$$12y(y) - 12y(4) = 0$$
$$12y(y-4) = 0$$
$$12y = 0 \quad \text{or} \quad y - 4 = 0$$
$$y = 0 \quad \text{or} \quad y = 4$$

The solutions are 0 and 4.

18. $3x^2 - 5x + 1 = 0$

Use the quadratic formula.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$$
$$= \frac{5 \pm \sqrt{25 - 12}}{6}$$
$$x = \frac{5 + \sqrt{13}}{6} \quad \text{or} \quad x = \frac{5 - \sqrt{13}}{6}$$
$$\approx 1.434 \qquad \approx .232$$

The solutions are $\frac{5+\sqrt{13}}{6} \approx 1.434$ and $\frac{5-\sqrt{13}}{6} \approx .232$.

19.
$$2m^2 = m + 4$$
$$2m^2 - m - 4 = 0$$

Use the quadratic formula.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-4)}}{2(2)}$$
$$x = \frac{1 \pm \sqrt{1 + 32}}{4}$$
$$x = \frac{1 \pm \sqrt{33}}{4}$$

The solutions are $\frac{1+\sqrt{33}}{4} \approx 1.686$ and $\frac{1-\sqrt{33}}{4} \approx -1.186$.

20. $p^2 + p - 1 = 0$

$$p = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{5}}{2}$$

The solutions are $\frac{-1+\sqrt{5}}{2} \approx .618$ and $\frac{-1-\sqrt{5}}{2} \approx -1.618$.

21.
$$k^2 - 10k = 20$$
$$k^2 - 10k + 20 = 0$$

$$k = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(20)}}{2(1)}$$
$$k = \frac{10 \pm \sqrt{100 - 80}}{2}$$
$$k = \frac{10 \pm \sqrt{20}}{2}$$
$$k = \frac{10 \pm \sqrt{4}\sqrt{5}}{2}$$
$$k = \frac{10 \pm 2\sqrt{5}}{2}$$
$$k = \frac{2(5 \pm 2\sqrt{5})}{2}$$
$$k = 5 \pm \sqrt{5}$$

The solutions are $5 + \sqrt{5} \approx 7.236$ and $5 - \sqrt{5} \approx 2.764$.

22. $2x^2 + 12x + 5 = 0$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(2)(5)}}{2(2)}$$
$$= \frac{-12 \pm \sqrt{104}}{4} = \frac{-12 \pm \sqrt{4 \cdot 26}}{4}$$
$$= \frac{-12 \pm \sqrt{4}\sqrt{26}}{4} = \frac{-12 \pm 2\sqrt{26}}{4}$$
$$= \frac{2(-6 \pm \sqrt{26})}{2 \cdot 2} = \frac{-6 \pm \sqrt{26}}{2}$$

The solutions are $\frac{-6+\sqrt{26}}{2} \approx -.450$ and $\frac{-6-\sqrt{26}}{2} \approx -5.550$.

23.
$$2r^2 - 7r + 5 = 0$$
$$(2r - 5)(r - 1) = 0$$
$$2r - 5 = 0 \quad \text{or} \quad r - 1 = 0$$
$$2r = 5$$
$$r = \frac{5}{2} \quad \text{or} \quad r = 1$$

The solutions are $\frac{5}{2}$ and 1.

24. $2x^2 - 7x + 30 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(30)}}{2(2)}$$
$$x = \frac{7 \pm \sqrt{49 - 240}}{4}$$
$$x = \frac{7 \pm \sqrt{-191}}{4}$$

Since there is a negative number under the radical sign, $\sqrt{-191}$ is not a real number. Thus, there are no real-number solutions.

25.
$$3k^2 + k = 6$$
$$3k^2 + k - 6 = 0$$
$$k = \frac{-1 \pm \sqrt{1 - 4(3)(-6)}}{2(3)}$$
$$= \frac{-1 \pm \sqrt{73}}{6}$$

The solutions are $\frac{-1+\sqrt{73}}{6} \approx 1.257$ and $\frac{-1-\sqrt{73}}{6} \approx -1.591$.

26. $5m^2 + 5m = 0$

$$5m(m + 1) = 0$$
$$5m = 0 \quad \text{or} \quad m + 1 = 0$$
$$m = 0 \quad \text{or} \quad m = -1$$

The solutions are 0 and -1.

27.
$$\frac{3x - 2}{7} = \frac{x + 2}{5}$$
$$35\left(\frac{3x - 2}{7}\right) = 35\left(\frac{x + 2}{5}\right)$$
$$5(3x - 2) = 7(x + 2)$$
$$15x - 10 = 7x + 14$$
$$8x = 24$$
$$x = 3$$

28. $\frac{x}{3} - 7 = 6 - \frac{3x}{4}$

Multiply both sides by 12, the least common denominator of 3 and 4.

$$12\left(\frac{x}{3} - 7\right) = 12\left(6 - \frac{3x}{4}\right)$$
$$12\left(\frac{x}{3}\right) - (12)(7) = (12)(6) - (12)\left(\frac{3x}{4}\right)$$
$$4x - 84 = 72 - 9x$$
$$13x - 84 = 72$$
$$13x = 156$$
$$x = 12$$

The solution is 12.

29. $\frac{4}{x - 3} - \frac{8}{2x + 5} + \frac{3}{x - 3} = 0$

$$\frac{4}{x - 3} + \frac{3}{x - 3} - \frac{8}{2x + 5} = 0$$
$$\frac{7}{x - 3} - \frac{8}{2x + 5} = 0$$

Multiply both sides by $(x - 3)(2x + 5)$. Note that $x \neq 3$ and $x \neq -\frac{5}{2}$.

$$(x-3)(2x+5)\left(\frac{7}{x - 3} - \frac{8}{2x + 5}\right) = (x-3)(2x+5)(0)$$
$$7(2x + 5) - 8(x - 3) = 0$$
$$14x + 35 - 8x + 24 = 0$$
$$6x + 59 = 0$$
$$6x = -59$$
$$x = -\frac{59}{6}$$

Note: It is especially important to check solutions of equations that involve rational expressions. Here, a check shows that $-\frac{59}{6}$ is a solution.

30. $\dfrac{5}{2p+3} - \dfrac{3}{p-2} = \dfrac{4}{2p+3}$

Multiply both sides by $(2p+3)(p-2)$.

Note that $p \neq -\frac{3}{2}$ and $p \neq 2$.

$$(2p+3)(p-2)\left(\frac{5}{2p+3} - \frac{3}{p-2}\right) = (2p+3)(p-2)\left(\frac{4}{2p+3}\right)$$

$$(2p+3)(p-2)\left(\frac{5}{2p+3}\right) - (2p+3)(p-2)\left(\frac{3}{p-2}\right) = (2p+3)(p-2)\left(\frac{4}{2p+3}\right)$$

$$\begin{aligned}(p-2)(5) - (2p+3)(3) &= (p-2)(4)\\ 5p - 10 - 6p - 9 &= 4p - 8\\ -p - 19 &= 4p - 8\\ -5p - 19 &= -8\\ -5p &= 11\\ p &= -\frac{11}{5}\end{aligned}$$

The solutions is $-\frac{11}{5}$.

31. $\dfrac{2}{m} + \dfrac{m}{m+3} = \dfrac{3m}{m^2+3m}$

$\dfrac{2}{m} + \dfrac{m}{m+3} = \dfrac{3m}{m(m+3)}$

Multiply both sides by $m(m+3)$.
Note that $m \neq 0$, and $m \neq -3$.

$$m(m+3)\left(\frac{2}{m} + \frac{m}{m+3}\right) = m(m+3)\left(\frac{3m}{m(m+3)}\right)$$

$$\begin{aligned}2(m+3) + m(m) &= 3m\\ 2m + 6 + m^2 &= 3m\\ m^2 - m + 6 &= 0\end{aligned}$$

$$\begin{aligned}m &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(6)}}{2(1)}\\ &= \frac{1 \pm \sqrt{1-24}}{2}\\ &= \frac{1 \pm \sqrt{-23}}{2}\end{aligned}$$

There are no real number solutions.

32. $\dfrac{2y}{y-1} = \dfrac{5}{y} + \dfrac{10-8y}{y^2-y}$

$\dfrac{2y}{y-1} = \dfrac{5}{y} + \dfrac{10-8y}{y(y-1)}$

Multiply both sides by $y(y-1)$.
Note that $y \neq 0$ and $y \neq 1$.

$$y(y-1)\left(\frac{2y}{y-1}\right) = y(y-1)\left[\frac{5}{y} + \frac{10-8y}{y(y-1)}\right]$$

$$y(y-1)\left(\frac{2y}{y-1}\right) = y(y-1)\left(\frac{5}{y}\right) + y(y-1)\left[\frac{10-8y}{y(y-1)}\right]$$

$$\begin{aligned}y(2y) &= (y-1)(5) + (10-8y)\\ 2y^2 &= 5y - 5 + 10 - 8y\\ 2y^2 &= 5 - 3y\\ 2y^2 + 3y - 5 &= 0\\ (2y+5)(y-1) &= 0\end{aligned}$$

$$2y + 5 = 0 \quad \text{or} \quad y - 1 = 0$$

$$y = -\frac{5}{2} \quad \text{or} \quad y = 1$$

Since $y \neq 1$, 1 is not a solution.
The solution is $-\frac{5}{2}$.

33. $\dfrac{1}{x-2} - \dfrac{3x}{x-1} = \dfrac{2x+1}{x^2-3x+2}$

$\dfrac{1}{x-2} - \dfrac{3x}{x-1} = \dfrac{2x+1}{(x-2)(x-1)}$

Multiply both sides by $(x-2)(x-1)$.
Note that $x \neq 2$ and $x \neq 1$.

$$(x-2)(x-1)\left(\frac{1}{x-2} - \frac{3x}{x-1}\right) = (x-2)(x-1) \cdot \left[\frac{2x+1}{(x-2)(x-1)}\right]$$

$$(x-2)(x-1)\left(\frac{1}{x-2}\right) - (x-2)(x-1)\cdot\left(\frac{3x}{x-1}\right) = \frac{(x-2)(x-2)(2x+1)}{(x-2)(x-1)}$$

$$\begin{aligned}(x-1) - (x-2)(3x) &= 2x + 1\\ x - 1 - 3x^2 + 6x &= 2x + 1\\ -3x^2 + 7x - 1 &= 2x + 1\\ -3x^2 + 5x - 2 &= 0\\ 3x^2 - 5x + 2 &= 0\\ (3x-2)(x-1) &= 0\end{aligned}$$

$$3x - 2 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = \frac{2}{3} \quad \text{or} \quad x = 1$$

1 is not a solution since $x \neq 1$.
The solution is $\frac{2}{3}$.

34. $$\frac{5}{a} + \frac{-7}{a+1} = \frac{a^2 - 2a + 4}{a^2 + a}$$

$$a(a+1)\left(\frac{5}{a} + \frac{-7}{a+1}\right) = a(a+1)\left(\frac{a^2 - 2a + 4}{a^2 + a}\right)$$

Note that $a \neq 0$ and $a \neq -1$.

$$\begin{aligned} 5(a+1) + (-7)(a) &= a^2 - 2a + 4 \\ 5a + 5 - 7a &= a^2 - 2a + 4 \\ 5 - 2a &= a^2 - 2a + 4 \\ 5 &= a^2 + 4 \\ 0 &= a^2 - 1 \\ 0 &= (a+1)(a-1) \end{aligned}$$

$$a + 1 = 0 \quad \text{or} \quad a - 1 = 0$$
$$a = -1 \quad \text{or} \quad a = 1$$

Since -1 would make two denominators zero, 1 is the only solution.

35. $$\frac{2b^2 + 5b - 8}{b^2 + 2b} + \frac{5}{b+2} = -\frac{3}{b}$$
$$\frac{2b^2 + 5b - 8}{b(b+2)} + \frac{5}{b+2} = \frac{-3}{b}$$

Multiply both sides by $b(b+2)$.
Note that $b \neq 0$ and $b \neq -2$.

$$b(b+2)\left(\frac{2b^2 + 5b - 8}{b^2 + 2b}\right) + b(b+2)\left(\frac{5}{b+2}\right) = b(b+2)\left(-\frac{3}{b}\right)$$

$$\begin{aligned} 2b^2 + 5b - 8 + 5b &= (b+2)(-3) \\ 2b^2 + 10b - 8 &= -3b - 6 \\ 2b^2 + 13b - 2 &= 0 \end{aligned}$$

$$b = \frac{-(-13) \pm \sqrt{(13)^2 - 4(2)(-2)}}{2(2)} = \frac{-13 \pm \sqrt{169 + 16}}{4}$$

$$b = \frac{-13 \pm \sqrt{185}}{4}$$

The solutions are $\frac{-13+\sqrt{185}}{4} \approx .150$ and $\frac{-13-\sqrt{185}}{4} \approx -6.650$.

36. $$\frac{2}{x^2 - 2x - 3} + \frac{5}{x^2 - x - 6} = \frac{1}{x^2 + 3x + 2}$$
$$\frac{2}{(x-3)(x+1)} + \frac{5}{(x-3)(x+2)} = \frac{1}{(x+2)(x+1)}$$

Multiply both sides by $(x-3)(x+1)(x+2)$.
Note that $x \neq 3$, $x \neq -1$, and $x \neq -2$.

$$(x-3)(x+1)(x+2)\left(\frac{2}{(x-3)(x+1)}\right) + (x-3)(x+1)(x+2)\left(\frac{5}{(x-3)(x+2)}\right) = (x-3)(x+1)(x+2)\left(\frac{1}{(x+2)(x+1)}\right)$$

$$\begin{aligned} 2(x+2) + 5(x+1) &= x - 3 \\ 2x + 4 + 5x + 5 &= x - 3 \\ 7x + 9 &= x - 3 \\ 6x + 9 &= -3 \\ 6x &= -12 \\ x &= -2 \end{aligned}$$

However, $x \neq -2$. Therefore there is no solution.

37. $$\frac{2}{y^2 + 7y + 12} - \frac{1}{y^2 + 5y + 6} = \frac{5}{y^2 + 6y + 8}$$
$$\frac{2}{(y+4)(y+3)} - \frac{1}{(y+3)(y+2)} = \frac{5}{(y+4)(y+2)}$$

Multiply both sides by $(y+4)(y+3)(y+2)$.
Note that $y \neq -4$, $y \neq -3$, and $y \neq -2$.

$$(y+4)(y+3)(y+2)\left(\frac{2}{(y+4)(y+3)}\right) - (y+4)(y+3)(y+2)\left(\frac{1}{(y+3)(y+2)}\right) = (y+4)(y+3)(y+2)\left(\frac{5}{(y+4)(y+2)}\right)$$

$$\begin{aligned} 2(y+2) - (y+4) &= 5(y+3) \\ 2y + 4 - y - 4 &= 5y + 15 \\ y &= 5y + 15 \\ -4y &= 15 \\ y &= -\frac{15}{4} \end{aligned}$$

The solution is $-\frac{15}{4}$.