R.5 Inequalities

1. $x < 0$

Because the inequality symbol means "less than," the endpoint at 0 is not included. This inequality is written in interval notation is $(-\infty, 0)$. To graph this interval on a number line, place an open circle at 0 and draw a heavy arrow pointing to the left.

2. $x \geq -3$

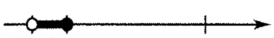
Because the inequality sign means "greater than or equal to," the endpoint at -3 is included. This inequality is written in interval notation as $[-3, \infty)$. To graph this interval on a number line, place a closed circle at -3 and draw a heavy arrow pointing to the right.

3. $-1 \leq x < 2$

The endpoint at -1 is included, but the endpoint at 2 is not. This inequality is written in interval notation as $[-1, 2)$. To graph this interval, place a closed circle at -1 and an open circle at 2; then draw a heavy line segment between them.

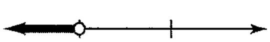
4. $-5 < x \leq -4$

The endpoint at -4 is included, but the endpoint at -5 is not. This inequality is written in interval notation as $(-5, 4]$. To graph this interval, place an open circle at -5 and a closed circle at 4; then draw a heavy line segment between them.



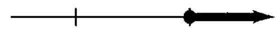
5. $-9 > x$

This inequality may be rewritten as $x < -9$, and is written in interval notation as $(-\infty, -9)$. Note that the endpoint at -9 is not included. To graph this interval, place an open circle at -9 and draw a heavy arrow pointing to the left.



6. $6 \leq x$

This inequality may be written as $x \geq 6$, and is written in interval notation as $[6, \infty)$. Note that the endpoint at 6 is included. To graph this interval, place a closed circle at 6 and draw a heavy arrow pointing to the right.



7. $(-4,\ 3)$

This represents all the numbers between -4 and 3, not including the endpoints. This interval can be written as the inequality $-4 < x < 3$.

8. $[2, 7)$

This represents all the numbers between 2 and 7, including 2 but not including 7. This interval can be written as the inequality $2 \leq x < 7$.

9. $(-\infty,\ -1]$

This represents all the numbers to the left of -1 on the number line and includes the endpoint. This interval can be written as the inequality $x \leq -1$.

10. $(3, \infty)$

This represents all the numbers to the right of 3, and does not include the endpoint. This interval can be written as the inequality $x > 3$.

11. Notice that the endpoint -2 is included, but 6 is not. The interval show in the graph can be written as the inequality $-2 \leq x < 6$.

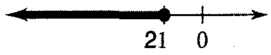
12. Notice that neither endpoint is included. The interval shown in the graph can be written as $0 < x < 8$.

13. Notice that both endpoints are included. The interval shown in the graph can be written as $x \leq -4$ or $x \geq 4$.

14. Notice that the endpoint 0 is not included, but 3 is included. The interval shown in the graph can be written as $x < 0$ or $x \geq 3$.

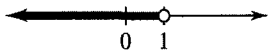
15.
$$-3p-2 \ge 1$$
$$-3p \ge 3$$
$$\left(-\frac{1}{3}\right)(-3p) \le \left(-\frac{1}{3}\right)(3)$$
$$p \le -1$$

The solution in interval notation is $(-\infty,\ -1]$.



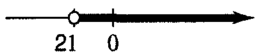
16.
$$6k-4 < 3k-1$$
$$6k < 3k+3$$
$$3k < 3$$
$$k < 1$$

The solution in interval notation is $(-\infty, 1)$.



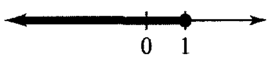
17.
$$m-(4+2m)+3 < 2m+2$$
$$m-4-2m+3 < 2m+2$$
$$-m-1 < 2m+2$$
$$-3m-1 < 2$$
$$-3m < 3$$
$$-\frac{1}{3}(-3m) > -\frac{1}{3}(3)$$
$$m > -1$$

The solution is $(-1,\ \infty)$.



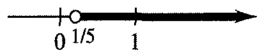
18.
$$-2(3y-8) \ge 5(4y-2)$$
$$-6y+16 \ge 20y-10$$
$$-6y+16+(-16) \ge 20y-10+(-16)$$
$$-6y \ge 20y-26$$
$$-6y+(-20y) \ge 20y-26$$
$$-26y \ge -26$$
$$-\frac{1}{26}(-26)y \le -\frac{1}{26}(-26)$$
$$y \le 1$$

The solution is $(-\infty, 1]$.



19.
$$3p-1 < 6p+2(p-1)$$
$$3p-1 < 6p+2p-2$$
$$3p-1 < 8p-2$$
$$-5p-1 < -2$$
$$-5p < -1$$
$$-\frac{1}{5}(-5p) > -\frac{1}{5}(-1)$$
$$p > \frac{1}{5}$$

The solution is $\left(\frac{1}{5},\ \infty\right)$.

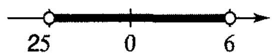


20.
$$x+5(x+1) > 4(2-x)+x$$
$$x+5x+5 > 8-4x+x$$
$$6x+5 > 8-3x$$
$$6x > 3-3x$$
$$9x > 3$$
$$x > \frac{1}{3}$$

The solution is $\left(\frac{1}{3}, \infty\right)$.

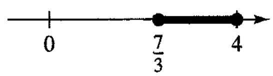
21.
$$-7 < y-2 < 4$$
$$-7+2 < y-2+2 < 4+2$$
$$-5 < y < 6$$

The solution is $(-5, 6)$.



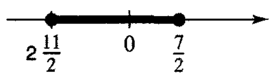
22.
$$8 \le 3r+1 \le 13$$
$$8+(-1) \le 3r+1+(-1) \le 13+(-1)$$
$$7 \le 3r \le 12$$
$$\frac{1}{3}(7) \le \frac{1}{3}(3r) \le \frac{1}{3}(12)$$
$$\frac{7}{3} \le r \le 4$$

The solution is $\left[\frac{7}{3}, 4\right]$.



23.
$$-4 \le \frac{2k-1}{3} \le 2$$
$$3(-4) \le 3\left(\frac{2k-1}{3}\right) \le 3(2)$$
$$-12 \le 2k-1 \le 6$$
$$-11 \le 2k \le 7$$
$$-\frac{11}{2} \le k \le \frac{7}{2}$$

The solution is $\left[-\frac{11}{2},\ \frac{7}{2}\right]$.

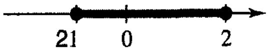


24. $-1 \le \dfrac{5y+2}{3} \le 4$

$$3(-1) \le 3\left(\frac{5y+2}{3}\right) \le 3(4)$$

$$-3 \le 5y + 2 \le 12$$
$$-5 \le 5y \le 10$$
$$-1 \le y \le 2$$

The solution is $[-1,\ 2]$.

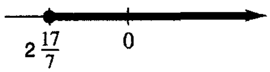


25. $\dfrac{3}{5}(2p+3) \ge \dfrac{1}{10}(5p+1)$

$$10\left(\frac{3}{5}\right)(2p+3) \ge 10\left(\frac{1}{10}\right)(5p+1)$$

$$6(2p+3) \ge 5p+1$$
$$12p + 18 \ge 5p + 1$$
$$7p \ge -17$$
$$p \ge -\frac{17}{7}$$

The solution is $[-\frac{17}{7},\ \infty)$.

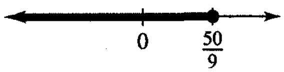


26. $\dfrac{8}{3}(z-4) \le \dfrac{2}{9}(3z+2)$

$$(9)\frac{8}{3}(z-4) \le (9)\frac{2}{9}(3z+2)$$

$$24(z-4) \le 2(3z+2)$$
$$24z - 96 \le 6z + 4$$
$$24z \le 6z + 100$$
$$18z \le 100$$
$$z \le \frac{100}{18}$$
$$z \le \frac{50}{9}$$

The solution is $(-\infty,\ \frac{50}{9}]$.



27. $(m+2)(m-4) < 0$

Solve $(m+2)(m-4) = 0$.

$$m = -2 \quad \text{or} \quad m = 4$$

Intervals: $(-\infty, -2),\ (-2, 4), (4, \infty)$

For $(-\infty, -2)$, choose -3 to test for m.

$$(-3+2)(-3-4) = -1(-7) = 8 \not< 0$$

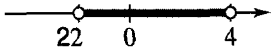
For $(-2,\ 4)$, choose 0.

$$(0+2)(0-4) = 2(-4) = -8 < 0$$

For $(4,\ \infty)$, choose 5.

$$(5+2)(5-4) = 7(1) = 7 \not< 0$$

The solution is $(-2,\ 4)$.



28. $(t+6)(t-1) \ge 0$

Solve $(t+6)(t-1) = 0$.

$$(t+6)(t-1) = 0$$
$$t = -6 \quad \text{or} \quad t = 1$$

Intervals: $(-\infty, -6),\ (-6, 1),\ (1, \infty)$

For $(-\infty, -6)$, choose -7 to test for t.

$$(-7+6)(-7-1) = (-1)(-8) = 8 \ge 0$$

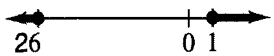
For $(-6,\ 1)$, choose 0.

$$(0+6)(0-1) = (6)(-1) = -6 \not\ge 0$$

For $(1,\ \infty)$, choose 2.

$$(2+6)(2-1) = (8)(1) = 8 \ge 0$$

Because the symbol $\ge$ is used, the endpoints -6 and 1 are included in the solution, $(-\infty, -6] \cup [1, \infty)$.



29. $y^2 - 3y + 2 < 0$

$(y-2)(y-1) < 0$

Solve $(y-2)(y-1) = 0$.

$$y = 2 \quad \text{or} \quad y = 1$$

Intervals: $(-\infty, 1)$, $(1,2), (2,\infty)$

For $(-\infty, 1)$, choose $y = 0$.

$$0^2 - 3(0) + 2 = 2 \not< 0$$

For $(1,2)$, choose $y = \frac{3}{2}$.

$$\left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 2 = \frac{9}{4} - \frac{9}{2} + 2$$
$$= \frac{9 - 18 + 8}{4}$$
$$= -\frac{1}{4} < 0$$

For $(2,\infty)$, choose 3.

$$3^2 - 3(3) + 2 = 2 \not< 0$$

The solution is $(1,2)$.

30. $2k^2 + 7k - 4 > 0$

Solve $2k^2 + 7k - 4 = 0$.

$$2k^2 + 7k - 4 = 0$$
$$(2k-1)(k+4) = 0$$
$$k = \frac{1}{2} \quad \text{or} \quad k = -4$$

Intervals: $(-\infty, -4), \left(-4, \frac{1}{2}\right) \left(\frac{1}{2}, \infty\right)$

For $(-\infty, -4)$, choose -5.

$$2(-5)^2 + 7(-5) - 4 = 11 > 0$$

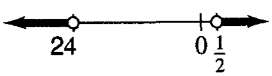
For $\left(-4, \frac{1}{2}\right)$, choose 0.

$$2(0)^2 + 7(0) - 4 = -4 \not> 0$$

For $\left(\frac{1}{2}, \infty\right)$, choose 1.

$$2(1)^2 + 7(1) - 4 = 5 > 0$$

The solution is $(-\infty, -4) \cup \left(\frac{1}{2}, \infty\right)$.



31. $q^2 - 7q + 6 \leq 0$

Solve $q^2 - 7q + 6 = 0$.

$$(q-1)(q-6) = 0$$
$$q = 1 \quad \text{or} \quad q = 6$$

These solutions are also solutions of the given inequality because the symbol $\leq$ indicates that the endpoints are included.

Intervals $(-\infty, 1)$, $(1,6)$, $(6,\infty)$

For $(-\infty, 1)$, choose 0.

$$0^2 - 7(0) + 6 = 6 \not\leq 0$$

For $(1,6)$, choose 2.

$$2^2 - 7(2) + 6 = -4 \leq 0$$

For $(6,\infty)$, choose 7.

$$7^2 - 7(7) + 6 = 6 \not\leq 0$$

The solution is $[1,6]$.

32. $2k^2 - 7k - 15 \leq 0$

Solve $2k^2 - 7k - 15 = 0$.

$$2k^2 - 7k - 15 = 0$$
$$(2k+3)(k-5) = 0$$
$$k = -\frac{3}{2} \quad \text{or} \quad k = 5$$

Intervals: $\left(-\infty, -\frac{3}{2}\right), \left(-\frac{3}{2}, 5\right), (5,\infty)$

For $\left(-\infty, -\frac{3}{2}\right)$, choose -2.

$$2(-2)^2 - 7(-2) - 15 = 7 \not\leq 0$$

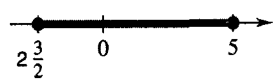
For $\left(-\frac{3}{2}, 5\right)$, choose 0.

$$2(0)^2 - 7(0) - 15 = -15 \leq 0$$

For $(5,\infty)$, choose 6.

$$2(6)^2 - 7(6) - 15 \not\leq 0$$

The solution is $\left[-\frac{3}{2}, 5\right]$.



33. $6m^2+m>1$

Solve $6m^2+m=1$.

$$6m^2+m=0$$
$$(2m+1)(3m-1)=0$$
$$m=-\frac{1}{2} \quad \text{or} \quad m=\frac{1}{3}$$

Intervals: $\left(-\infty,-\frac{1}{2}\right), \left(-\frac{1}{2},\frac{1}{3}\right), \left(\frac{1}{3},\infty\right)$

For $\left(-\infty,-\frac{1}{2}\right)$, choose -1.

$$6(-1)^2+(-1)=5>1$$

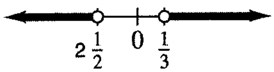
For $\left(-\frac{1}{2},\frac{1}{3}\right)$, choose 0.

$$6(0)^2+0=0\not>1$$

For $\left(\frac{1}{3},\infty\right)$ choose 1.

$$6(1)^2+1=7>1$$

The solution is $\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{1}{3},\infty\right)$.



34. $10r^2+r\le 2$

Solve $10r^2+r=2$.

$$10r^2+r=2$$
$$10r^2+r-2=0$$
$$(5r-2)(2r+1)=0$$
$$r=\frac{2}{5} \quad \text{or} \quad r=-\frac{1}{2}$$

Intervals: $\left(-\infty,-\frac{1}{2}\right), \left(-\frac{1}{2},\frac{2}{5}\right), \left(\frac{2}{5},\infty\right)$

For $\left(-\infty,-\frac{1}{2}\right)$, choose -1.

$$10(-1)^2+(-1)=9\not\le 2$$

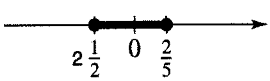
For $\left(-\frac{1}{2},\frac{2}{5}\right)$, choose 0.

$$10(0)^2+0=0\le 2$$

For $\left(\frac{2}{5},\infty\right)$, choose 1.

$$10(1)^2+1=11\not\le 2$$

The solution is $\left[-\frac{1}{2},\frac{2}{5}\right]$.



35. $2y^2+5y\le 3$

Solve $2y^2+5y=3$.

$$2y^2+5y-3=0$$
$$(y+3)(2y-1)=0$$
$$y=-3 \quad \text{or} \quad y=\frac{1}{2}$$

Intervals: $(-\infty,-3)$, $\left(-3,\frac{1}{2}\right)$, $\left(\frac{1}{2},\infty\right)$

For $(-\infty,-3)$, choose -4.

$$2(-4)^2+5(-4)=12\not\le 3$$

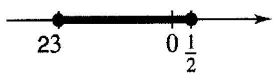
For $\left(-3,\frac{1}{2}\right)$, choose 0.

$$2(0)^2+5(0)=0\le 3$$

For $\left(\frac{1}{2},\infty\right)$, choose 1.

$$2(1)^2+5(1)=7\not\le 3$$

The solution is $\left[-3,\frac{1}{2}\right]$.



36. $3a^2+a>10$

Solve $3a^2+a=10$.

$$3a^2+a=10$$
$$3a^2+a-10=0$$
$$(3a-5)(a+2)=0$$
$$a=\frac{5}{3} \quad \text{or} \quad a=-2$$

Intervals: $(-\infty,-2)$, $\left(-2,\frac{5}{3}\right)$, $\left(\frac{5}{3},\infty\right)$

For $(-\infty,-2)$, choose -3.

$$3(-3)^2+(-3)=24>10$$

For $\left(-2,\frac{5}{3}\right)$, choose 0.

$$3(0)^2+0=0\not>10$$

For $\left(\frac{5}{3},\infty\right)$, choose 2.

$$3(2)^2+2=14>10$$

The solution is $(-\infty,-2)\cup\left(\frac{5}{3},\infty\right)$.