37. $x^2 \leq 25$

Solve $x^2 = 25$.

$$x = -5 \quad \text{or} \quad x = 5$$

Intervals: $(-\infty, -5)$, $(-5, 5)$, $(5, \infty)$

For $(-\infty, -5)$, choose -6.

$$(-6)^2 = 36 \nleq 25$$

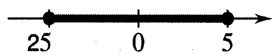
For $(-5, 5)$, choose 0.

$$0^2 = 0 \leq 25$$

For $(5, \infty)$, choose 6.

$$6^2 = 36 \nleq 25$$

The solution is $[-5, 5]$.



38. $p^2 - 16p > 0$

Solve $p^2 - 16p = 0$.

$$\begin{aligned} p^2 - 16p &= 0 \\ p(p - 16) &= 0 \\ p = 0 \quad \text{or} \quad p &= 16 \end{aligned}$$

Intervals: $(-\infty, 0)$, $(0, 16)$, $(16, \infty)$

For $(-\infty, 0)$, choose -1.

$$(-1)^2 - 16(-1) = 17 > 0$$

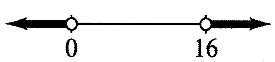
For $(0, 16)$, choose 1.

$$(1)^2 - 16(1) = -15 \ngtr 0$$

For $(16, \infty)$, choose 17.

$$(17)^2 - 16(17) = 17 > 0$$

The solution is $(-\infty, 0) \cup (16, \infty)$.



39. $\dfrac{m-3}{m+5} \leq 0$

Solve $\dfrac{m-3}{m+5} = 0$.

$$\begin{aligned} (m+5)\frac{m-3}{m+5} &= (m+5)(0) \\ m - 3 &= 0 \\ m &= 3 \end{aligned}$$

Set the denominator equal to 0 and solve.

$$\begin{aligned} m + 5 &= 0 \\ m &= -5 \end{aligned}$$

Intervals: $(-\infty, -5)$, $(-5, 3)$, $(3, \infty)$

For $(-\infty, -5)$, choose -6.

$$\frac{-6-3}{-6+5} = 9 \nleq 0$$

For $(-5, 3)$, choose 0.

$$\frac{0-3}{0+5} = -\frac{3}{5} \leq 0$$

For $(3, \infty)$, choose 4.

$$\frac{4-3}{4+5} = \frac{1}{9} \nleq 0$$

Although the $\leq$ symbol is used, including -5 in the solution would cause the denominator to be zero.
The solution is $(-5, 3]$.

40. $\dfrac{r+1}{r-1} > 0$

Solve the equation $\dfrac{r+1}{r-1} = 0$.

$$\begin{aligned} \frac{r+1}{r-1} &= 0 \\ (r-1)\frac{r+1}{r-1} &= (r-1)(0) \\ r + 1 &= 0 \\ r &= -1 \end{aligned}$$

Find the value for which the denominator equals zero.

$$\begin{aligned} r - 1 &= 0 \\ r &= 1 \end{aligned}$$

Intervals: $(-\infty, -1)$, $(-1, 1)$, $(1, \infty)$

For $(-\infty, -1)$, choose -2.

$$\frac{-2+1}{-2-1} = \frac{-1}{-3} = \frac{1}{3} > 0$$

For $(-1, 1)$, choose 0.

$$\frac{0+1}{0-1} = \frac{1}{-1} = -1 \ngtr 0$$

For $(1, \infty)$, choose 2.

$$\frac{2+1}{2-1} = \frac{3}{1} = 3 > 0$$

The solution is $(-\infty, -1) \cup (1, \infty)$.

41. $\dfrac{k-1}{k+2} > 1$

Solve $\dfrac{k-1}{k+2} = 1$.

$$\begin{aligned} k-1 &= k+2 \\ -1 &\neq 2 \end{aligned}$$

The equation has no solution.
Solve $k+2=0$.

$$k=-2$$

Intervals: $(-\infty,-2)$, $(-2,\infty)$

For $(-\infty,-2)$, choose -3.

$$\frac{-3-1}{-3+2} = 4 > 1$$

For $(-2,\infty)$, choose 0.

$$\frac{0-1}{0+2} = -\frac{1}{2} \not> 1$$

The solution is $(-\infty,-2)$.

42. $\dfrac{a-5}{a+2} < -1$

Solve the equation $\dfrac{a-5}{a+2} = -1$.

$$\begin{aligned} \frac{a-5}{a+2} &= -1 \\ a-5 &= -1(a+2) \\ a-5 &= -a-2 \\ 2a &= 3 \\ a &= \frac{3}{2} \end{aligned}$$

Set the denominator equal to zero and solve for a.

$$\begin{aligned} a+2 &= 0 \\ a &= -2 \end{aligned}$$

Intervals: $(-\infty,-2)$, $\left(-2,\frac{3}{2}\right)$, $\left(\frac{3}{2},\infty\right)$

For $(-\infty,-2)$, choose -3.

$$\frac{-3-5}{-3+2} = \frac{-8}{-1} = 8 \not< -1$$

For $\left(-2,\frac{3}{2}\right)$, choose 0.

$$\frac{0-5}{0+2} = \frac{-5}{2} = -\frac{5}{2} < -1$$

For $\left(\frac{3}{2},\infty\right)$, choose 2.

$$\frac{2-5}{2+2} = \frac{-3}{4} = -\frac{3}{4} \not< -1$$

The solution is $\left(-2,\frac{3}{2}\right)$.

43. $\dfrac{2y+3}{y-5} \le 1$

Solve $\dfrac{2y+3}{y-5} = 1$.

$$\begin{aligned} 2y+3 &= y-5 \\ y &= -8 \end{aligned}$$

Solve $y-5=0$.

$$y=5$$

Intervals: $(-\infty,-8)$, $(-8,5)$, $(5,\infty)$

For $(-\infty,,-8)$, choose $y=-10$.

$$\frac{2(-10)+3}{-10-5} = \frac{17}{15} \not\le 1$$

For $(-8,5)$, choose $y=0$.

$$\frac{2(0)+3}{0-5} = -\frac{3}{5} \le 1$$

For $(5,\infty)$, choose $y=6$.

$$\frac{2(6)+3}{6-5} = \frac{15}{1} \not\le 1$$

The solution is $[-8,5)$.

44. $\dfrac{a+2}{3+2a} \le 5$

For the equation $\dfrac{a+2}{3+2a} = 5$.

$$\begin{aligned} \frac{a+2}{3+2a} &= 5 \\ a+2 &= 5(3+2a) \\ a+2 &= 15+10a \\ -9a &= 13 \\ a &= -\frac{13}{9} \end{aligned}$$

Set the denominator equal to zero and solve for a.

$$\begin{aligned} 3+2a &= 0 \\ 2a &= -3 \\ a &= -\frac{3}{2} \end{aligned}$$

Intervals: $\left(-\infty,-\frac{3}{2}\right)$, $\left(-\frac{3}{2},-\frac{13}{9}\right)$, $\left(-\frac{13}{9},\infty\right)$

For $\left(-\infty,-\frac{3}{2}\right)$, choose -2.

$$\frac{-2+2}{3+2(-2)} = \frac{0}{-1} = 0 \le 5$$

For $\left(-\frac{3}{2},-\frac{13}{9}\right)$, choose -1.46.

$$\frac{-1.46+2}{3+2(-1.46)}=\frac{.54}{.08}=6.75\not\leq 5$$

For $\left(-\frac{13}{9},\infty\right)$, choose 0.

$$\frac{0+2}{3+2(0)}=\frac{2}{3}\leq 5$$

The value $-\frac{3}{2}$ cannot be included in the solution since it would make the denominator zero. The solution is $\left(-\infty,-\frac{3}{2}\right)\cup\left[-\frac{13}{9},\infty\right)$.

45. $\dfrac{7}{k+2}\geq\dfrac{1}{k+2}$

Solve $\dfrac{7}{k+2}=\dfrac{1}{k+2}$.

$$\frac{7}{k+2}-\frac{1}{k+2}=0$$
$$\frac{6}{k+2}=0$$

The equation has no solution.

Solve $k+2=0$.

$$k=-2$$

Intervals: $(-\infty,-2)$, $(-2,\infty)$

For $(-\infty,-2)$, choose $k=-3$.

$$\frac{6}{-3+2}=-6\not\geq 0$$

For $(-2,\infty)$, choose $k=0$.

$$\frac{6}{0+2}=3\geq 0$$

The solution is $(-2,\infty)$.

46. $\dfrac{5}{p+1}>\dfrac{12}{p+1}$

Solve the equation $\dfrac{5}{p+1}=\dfrac{12}{p+1}$.

$$\frac{5}{p+1}=\frac{12}{p+1}$$
$$5=12$$

The equation has no solution.
Set the denominator equal to zero and solve for p.

$$p+1=0$$
$$p=-1$$

Intervals: $(-\infty,-1)$, $(-1,\infty)$

For $(-\infty,-1)$, choose -2.

$\dfrac{5}{-2+1}=-5$ and $\dfrac{12}{-2+1}=-12$, so

$$\frac{5}{-2+1}>\frac{12}{-2+1}.$$

For $(-1,\infty)$, choose 0.

$\dfrac{5}{0+1}=5$ and $\dfrac{12}{0+1}=12$, so

$$\frac{5}{0+1}\not>\frac{12}{0+1}.$$

The solution is $(-\infty,-1)$.

47. $\dfrac{3x}{x^2-1}<2$

Solve

$$\frac{3x}{x^2-1}=2.$$
$$3x=2x^2-2$$
$$-2x^2+3x+2=0$$
$$(2x+1)(-x+2)=0$$
$$x=-\frac{1}{2}\quad\text{or}\quad x=2$$

Set $x^2-1=0$.

$$x=1\quad\text{or}\quad x=-1$$

Intervals: $(-\infty,-1)$, $\left(-1,-\frac{1}{2}\right)$, $\left(-\frac{1}{2},1\right)$, $(1,2)$, $(2,\infty)$

For $(-\infty,-1)$, choose $x=-2$.

$$\frac{3(-2)}{(-2)^2-1}=-\frac{6}{3}=-2<2$$

For $\left(-1,-\frac{1}{2}\right)$, choose $x=-\frac{3}{4}$.

$$\frac{3\left(-\frac{3}{4}\right)}{\left(-\frac{3}{4}\right)^2-1}=\frac{-\frac{9}{4}}{\frac{9}{16}-1}=\frac{36}{7}\not<2$$

For $\left(-\frac{1}{2},1\right)$, choose $x=0$.

$$\frac{3(0)}{0^2-1}=0<2$$

For $(1,2)$, choose $x=\frac{3}{2}$.

$$\frac{3\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)^2-1}=\frac{\frac{9}{2}}{\frac{5}{4}}=\frac{18}{5}\not<2$$

For $(2,\infty)$, choose $x=3$.

$$\frac{3(3)}{3^2-1}=\frac{9}{8}<2$$

The solution is $(-\infty,-1)\cup\left(-\frac{1}{2},1\right)\cup(2,\infty)$.

48. $\dfrac{8}{p^2+2p} > 1$

Solve the equation $\dfrac{8}{p^2+2p} = 1.$

$$\begin{aligned}\frac{8}{p^2+2p} &= 1\\ 8 &= p^2+2p\\ 0 &= p^2+2p-8\\ 0 &= p+4)(p-2)\end{aligned}$$

$$p+4=0 \quad \text{or} \quad p-2=0$$
$$p=-4 \quad \text{or} \quad p=2$$

Set the denominator equal to zero and solve for p.

$$p^2+2p=0$$
$$p(p+2)=0$$
$$p=0 \quad \text{or} \quad p+2=0$$
$$p=-2$$

Intervals: $(-\infty,-4)$, $(-4,-2)$, $(-2,0)$, $(0,2)$, $(2,\infty)$

For $(-\infty,-4)$, choose -5.

$$\frac{8}{(-5)^2+2(-5)} = \frac{8}{15} \not> 1$$

For $(-4,-2)$, choose -3.

$$\frac{8}{(-3)^2+2(-3)} = \frac{8}{9-6} = \frac{8}{3} > 1$$

For $(-2,0)$, choose -1.

$$\frac{8}{(-1)^2+2(-1)} = \frac{8}{-1} = -8 \not> 1$$

For $(0,2)$, choose 1.

$$\frac{8}{(1)^2+2(1)} = \frac{8}{3} > 1$$

For $(2,\infty)$, choose 3.

$$\frac{8}{(3)^2+(2)(3)} = \frac{8}{15} \not> 1$$

The solution is $(-4,-2)\cup(0,2)$.

49. $\dfrac{z^2+z}{z^2-1} \geq 3$

Solve

$$\frac{z^2+z}{z^2-1} = 3.$$

$$\begin{aligned}z^2+z &= 3z^2-3\\ -2z^2+z+3 &= 0\\ -1(2z^2-z-3) &= 0\\ -1(z+1)(2z-3) &= 0\end{aligned}$$

$$z=-1 \quad \text{or} \quad z=\frac{3}{2}$$

Set $z^2-1=0$.

$$z^2=1$$
$$z=-1 \quad \text{or} \quad z=1$$

Intervals: $(-\infty,-1)$, $(-1,1)$, $\left(1,\frac{3}{2}\right)$, $\left(\frac{3}{2},\infty\right)$

For $(-\infty,-1)$, choose $x=-2$.

$$\frac{(-2)^2+3}{(-2)^2-1} = \frac{7}{3} \not\geq 3$$

For $(-1,1)$, choose $x=0$.

$$\frac{0^2+3}{0^2-1} = -3 \not\geq 3$$

For $\left(1,\frac{3}{2}\right)$, choose $x=\frac{3}{2}$.

$$\frac{\left(\frac{3}{2}\right)^2+3}{\left(\frac{3}{2}\right)^2-1} = \frac{21}{5} \geq 3$$

For $\left(\frac{3}{2},\infty\right)$, choose $x=2$.

$$\frac{2^2+3}{2^2-1} = \frac{7}{3} \not\geq 3$$

The solution is $\left(1,\frac{3}{2}\right]$.

50. $\dfrac{a^2+2a}{a^2-4} \leq 2$

Solve the equation $\dfrac{a^2+2a}{a^2-4} = 2.$

$$\begin{aligned}\frac{a^2+2a}{a^2-4} &= 2\\ a^2+2a &= 2(a^2-4)\\ a^2+2a &= 2a^2-8\\ 0 &= a^2-2a-8\\ 0 &= (a-4)(a+2)\end{aligned}$$

$$a-4=0 \quad \text{or} \quad a+2=0$$
$$a=4 \quad \text{or} \quad a=-2$$

But -2 is not a possible solution.
Set the denominator equal to zero and solve for a.

$$a^2-4=0$$
$$(a+2)(a-2)=0$$
$$a+2=0 \quad \text{or} \quad a-2=0$$
$$a=-2 \quad \text{or} \quad a=2$$

Intervals: $(-\infty,-2)$, $(-2,2)$,
$(2,4)$, $(4,\infty)$

For $(-\infty,-2)$, choose -3.

$$\frac{(-3)^2+2(-3)}{(-3)^2-4}=\frac{9-6}{9-4}=\frac{3}{5}\leq 2$$

For $(-2,2)$, choose 0.

$$\frac{(0)^2+2(0)}{0-4}=\frac{0}{-4}=0\leq 2$$

For $(2,4)$, choose 3.

$$\frac{(3)^2+2(3)}{(3)^2-4}=\frac{9+6}{9-5}=\frac{15}{4}\not\leq 2$$

For $(4,\infty)$, choose 5.

$$\frac{(5)^2+2(5)}{(5)^2-4}=\frac{25+10}{25-4}=\frac{35}{21}\leq 2$$

The value 4 will satisfy the original inequality, but the values -2 and 2 will not since they make the denominator zero. The solution is $(-\infty,-2)\cup(-2,2)\cup[4,\infty)$.