

47. $\tan \theta = \frac{\sqrt{7}}{3}$, $\sec \theta = \frac{4}{3}$. Then $\cos \theta = \frac{3}{4}$ and $\sin \theta = \tan \theta \cdot \cos \theta = \frac{\sqrt{7}}{4}$, $\csc \theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$, and $\cot \theta = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$.

49. $\sin \theta = \frac{3}{5}$. Since $\cos \theta < 0$, θ is in quadrant II. Thus, $x = -\sqrt{5^2 - 3^2} = -\sqrt{16} = -4$. Therefore, $\cos \theta = -\frac{4}{5}$, $\tan \theta = -\frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = -\frac{5}{4}$, $\cot \theta = -\frac{4}{3}$.

51. $\tan \theta = -\frac{1}{2}$. $\sec^2 \theta = 1 + \tan^2 \theta = 1 + \frac{1}{4} = \frac{5}{4} \Leftrightarrow \cos^2 \theta = \frac{4}{5} \Rightarrow \cos \theta = -\sqrt{\frac{4}{5}} = -\frac{2}{\sqrt{5}}$
 since $\cos \theta < 0$ in quadrant II. But $\tan \theta = \frac{\sin \theta}{\cos \theta} = -\frac{1}{2} \Leftrightarrow \sin \theta = -\frac{1}{2} \cos \theta$
 $= -\frac{1}{2} \left(-\frac{2}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}}$. Therefore, $\sin \theta + \cos \theta = \frac{1}{\sqrt{5}} + \left(-\frac{2}{\sqrt{5}}\right) = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$.

53. By the Pythagorean identity, $\sin^2 \theta + \cos^2 \theta = 1$ for any angle θ .

55. $\angle B = 180^\circ - 30^\circ - 80^\circ = 70^\circ$, and so by the Law of Sines, $x = \frac{10 \cdot \sin 30^\circ}{\sin 70^\circ} \approx 5.32$

57. $x^2 = 100^2 + 210^2 - 2 \cdot 100 \cdot 210 \cdot \cos 40^\circ \approx 21926.133 \Leftrightarrow x \approx 148.07$

59. $\sin B = \frac{20 \cdot \sin 60^\circ}{70} \approx 0.247 \Leftrightarrow \angle B \approx \sin^{-1} 0.247 \approx 14.33^\circ$. Then

$\angle C \approx 180^\circ - 60^\circ - 14.33^\circ = 105.67^\circ$, and so $x \approx \frac{70 \cdot \sin 105.67^\circ}{\sin 60^\circ} \approx 77.82$.