1. $\log_2[x(x-1)] = \log_2 x + \log_2(x-1)$

3. $\log 7^{23} = 23 \log 7$

5. $\log_2(AB^2) = \log_2 A + \log_2 B^2 = \log_2 A + 2\log_2 B$

7. $\log_3(x\sqrt{y}) = \log_3 x + \log_3 \sqrt{y} = \log_3 x + \frac{1}{2}\log_3 y$

9. $\log_5 \sqrt[3]{x^2+1} = \frac{1}{3}\log_5(x^2+1)$

11. $\ln\sqrt{ab} = \frac{1}{2}\ln ab = \frac{1}{2}(\ln a + \ln b)$

13. $\log\left(\dfrac{x^3y^4}{z^6}\right) = \log\left(x^3y^4\right) - \log z^6 = 3\log x + 4\log y - 6\log z$

15. $\log_2\left(\dfrac{x(x^2+1)}{\sqrt{x^2-1}}\right) = \log_2 x + \log_2(x^2+1) - \dfrac{1}{2}\log_2(x^2-1)$

17. $\ln\left(x\sqrt{\dfrac{y}{z}}\right) = \ln x + \dfrac{1}{2}\ln\left(\dfrac{y}{z}\right) = \ln x + \dfrac{1}{2}(\ln y - \ln z)$

19. $\log\sqrt[4]{x^2+y^2} = \frac{1}{4}\log(x^2+y^2)$

21. $\log\sqrt{\dfrac{x^2+4}{(x^2+1)(x^3-7)^2}} = \frac{1}{2}\log\dfrac{x^2+4}{(x^2+1)(x^3-7)^2} = \frac{1}{2}[\log(x^2+4) - \log(x^2+1)(x^3-7)^2]$

$= \frac{1}{2}[\log(x^2+4) - \log(x^2+1) - 2\log(x^3-7)]$

23. $\ln\dfrac{z^4\sqrt{x}}{\sqrt[3]{y^2+6y+17}} = \ln\left(z^4\sqrt{x}\right) - \ln\sqrt[3]{y^2+6y+17} = 4\ln z + \dfrac{1}{2}\ln x - \dfrac{1}{3}\ln(y^2+6y+17)$

25. $\log_5\sqrt{125} = \log_5 5^{3/2} = \frac{3}{2}$

27. $\log 2 + \log 5 = \log 10 = 1$

29. $\log_4 192 - \log_4 3 = \log_4 \frac{192}{3} = \log_4 64 = \log_4 4^3 = 3$

31. $\ln 6 - \ln 15 + \ln 20 = \ln\frac{6}{15} + \ln 20 = \ln(\frac{2}{5}\cdot 20) = \ln 8$

33. $10^{2\log 4} = (10^{\log 4})^2 = 4^2 = 16$

35. $\log_3 5 + 5\log_3 2 = \log_3 5 + \log_3 2^5 = \log_3(5\cdot 2^5) = \log_3 160$

37. $\log_2 A + \log_2 B - 2\log_2 C = \log_2(AB) - \log_2(C^2) = \log_2\left(\dfrac{AB}{C^2}\right)$

39. $4\log x - \frac{1}{3}\log(x^2+1) + 2\log(x-1) = \log x^4 - \log\sqrt[3]{x^2+1} + \log(x-1)^2$

$= \log\left(\dfrac{x^4}{\sqrt[3]{x^2+1}}\right) + \log(x-1)^2 = \log\left(\dfrac{x^4(x-1)^2}{\sqrt[3]{x^2+1}}\right)$