

33.  $f(x) = \frac{1}{x}$ , has domain  $\{x \mid x \neq 0\}$ ;  $g(x) = 2x + 4$ , has domain  $(-\infty, \infty)$ .

$$(f \circ g)(x) = f(2x + 4) = \frac{1}{2x + 4}. \quad (f \circ g)(x) \text{ is defined for } 2x + 4 \neq 0 \Leftrightarrow x \neq -2. \text{ So the domain is } \{x \mid x \neq -2\} = (-\infty, -2) \cup (-2, \infty).$$

$$(g \circ f)(x) = g\left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right) + 4 = \frac{2}{x} + 4, \text{ the domain is } \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty).$$

$$(f \circ f)(x) = f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = x. \quad (f \circ f)(x) \text{ is defined whenever both } f(x) \text{ and } f(f(x)) \text{ are}$$

defined; that is, whenever  $\{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty)$ .

$$(g \circ g)(x) = g(2x + 4) = 2(2x + 4) + 4 = 4x + 8 + 4 = 4x + 12, \text{ and the domain is } (-\infty, \infty).$$

35.  $f(x) = |x|$ , has domain  $(-\infty, \infty)$ ;  $g(x) = 2x + 3$ , has domain  $(-\infty, \infty)$

$$(f \circ g)(x) = f(2x + 3) = |2x + 3|, \text{ and the domain is } (-\infty, \infty).$$

$$(g \circ f)(x) = g(|x|) = 2|x| + 3, \text{ and the domain is } (-\infty, \infty).$$

$$(f \circ f)(x) = f(|x|) = ||x|| = |x|, \text{ and the domain is } (-\infty, \infty).$$

$$(g \circ g)(x) = g(2x + 3) = 2(2x + 3) + 3 = 4x + 6 + 3 = 4x + 9. \text{ Domain is } (-\infty, \infty).$$

37.  $f(x) = \frac{x}{x+1}$ , has domain  $\{x \mid x \neq -1\}$ ;  $g(x) = 2x - 1$ , has domain  $(-\infty, \infty)$

$$(f \circ g)(x) = f(2x - 1) = \frac{2x - 1}{(2x - 1) + 1} = \frac{2x - 1}{2x}, \text{ and the domain is } \{x \mid x \neq 0\} = (-\infty, 0) \cup (0, \infty).$$

$$(g \circ f)(x) = g\left(\frac{x}{x+1}\right) = 2\left(\frac{x}{x+1}\right) - 1 = \frac{2x}{x+1} - 1, \text{ and the domain is } \{x \mid x \neq -1\} = (-\infty, -1) \cup (-1, \infty)$$

$$(f \circ f)(x) = f\left(\frac{x}{x+1}\right) = \frac{\frac{x}{x+1}}{\frac{x}{x+1} + 1} \cdot \frac{x+1}{x+1} = \frac{x}{x+x+1} = \frac{x}{2x+1}. \quad (f \circ f)(x) \text{ is defined}$$

whenever both  $f(x)$  and  $f(f(x))$  are defined; that is, whenever  $\{x \mid x \neq -1\}$  and  $2x + 1 \neq 0 \Rightarrow \{x \mid x \neq -\frac{1}{2}\}$  which is  $(-\infty, -1) \cup (-1, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$ .

$$(g \circ g)(x) = g(2x - 1) = 2(2x - 1) - 1 = 4x - 2 - 1 = 4x - 3, \text{ and the domain is } (-\infty, \infty).$$

39.  $f(x) = \sqrt[3]{x}$ , has domain  $(-\infty, \infty)$ ;  $g(x) = \sqrt[4]{x}$ , has domain  $[0, \infty)$ .

$$(f \circ g)(x) = f(\sqrt[4]{x}) = \sqrt[3]{\sqrt[4]{x}} = \sqrt[12]{x}. \quad (f \circ g)(x) \text{ is defined whenever both } g(x) \text{ and } f(g(x)) \text{ are defined. Since } f(x) \text{ has no restriction, the domain is } [0, \infty).$$

$$(g \circ f)(x) = g(\sqrt[3]{x}) = \sqrt[4]{\sqrt[3]{x}} = \sqrt[12]{x}. \quad (g \circ f)(x) \text{ is defined whenever both } f(x) \text{ and } g(f(x)) \text{ are defined; that is, whenever } x \geq 0. \text{ So the domain is } [0, \infty).$$

$$(f \circ f)(x) = f(\sqrt[3]{x}) = \sqrt[3]{\sqrt[3]{x}} = \sqrt[9]{x}. \quad (f \circ f)(x) \text{ is defined whenever both } f(x) \text{ and } f(f(x)) \text{ are defined. Since } f(x) \text{ is defined everywhere, the domain is } (-\infty, \infty).$$