

$(f \circ f)(x) = f(x^2) = (x^2)^2 = x^4$ , and the domain is  $(-\infty, \infty)$ .

$(g \circ g)(x) = g(\sqrt{x-3}) = \sqrt{\sqrt{x-3}-3}$ . For the domain we must have  $\sqrt{x-3} \geq 3 \Rightarrow x-3 \geq 9 \Rightarrow x \geq 12$ , so the domain is  $[12, \infty)$ .

36.  $f(x) = x - 4$  has domain  $(-\infty, \infty)$ .  $g(x) = |x + 4|$  has domain  $(-\infty, \infty)$ .

$(f \circ g)(x) = f(|x + 4|) = |x + 4| - 4$ , and the domain is  $(-\infty, \infty)$ .

$(g \circ f)(x) = g(x - 4) = |(x - 4) + 4| = |x|$ , and the domain is  $(-\infty, \infty)$ .

$(f \circ f)(x) = f(x - 4) = (x - 4) - 4 = x - 8$ , and the domain is  $(-\infty, \infty)$ .

$(g \circ g)(x) = g(|x + 4|) = ||x + 4| + 4| = |x + 4| + 4$  ( $|x + 4| + 4$  is always positive). The domain is  $(-\infty, \infty)$ .

38.  $f(x) = \frac{1}{\sqrt{x}}$  has domain  $\{x \mid x > 0\}$ ;  $g(x) = x^2 - 4x$  has domain  $(-\infty, \infty)$ .

$(f \circ g)(x) = f(x^2 - 4x) = \frac{1}{\sqrt{x^2 - 4x}}$ .  $(f \circ g)(x)$  is defined whenever  $0 < x^2 - 4x = x(x - 4)$ .

The product of two numbers is positive either when both numbers are negative or when both numbers are positive. So the domain of  $f \circ g$  is  $\{x \mid x < 0 \text{ and } x < 4\} \cup \{x \mid x > 0 \text{ and } x > 4\}$  which is  $(-\infty, 0) \cup (4, \infty)$ .

$(g \circ f)(x) = g\left(\frac{1}{\sqrt{x}}\right) = \left(\frac{1}{\sqrt{x}}\right)^2 - 4\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{x} - \frac{4}{\sqrt{x}}$ .  $(g \circ f)(x)$  is defined whenever both  $f(x)$  and  $g(f(x))$  are defined, that is, whenever  $x > 0$ . So the domain of  $g \circ f$  is  $(0, \infty)$ .

$(f \circ f)(x) = f\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\sqrt{\frac{1}{\sqrt{x}}}} = x^{1/4}$ .  $(f \circ f)(x)$  is defined whenever both  $f(x)$  and  $f(f(x))$

are defined, that is, whenever  $x > 0$ . So the domain of  $f \circ f$  is  $(0, \infty)$ .

$(g \circ g)(x) = g(x^2 - 4x) = (x^2 - 4x)^2 - 4(x^2 - 4x) = x^4 - 8x^3 + 16x^2 - 4x^2 + 16x = x^4 - 8x^3 + 12x^2 + 16x$ , and the domain is  $(-\infty, \infty)$ .

40.  $f(x) = \frac{2}{x}$  has domain  $\{x \mid x \neq 0\}$ ;  $g(x) = \frac{x}{x+2}$  has domain  $\{x \mid x \neq -2\}$ .

$(f \circ g)(x) = f\left(\frac{x}{x+2}\right) = \frac{2}{\frac{x}{x+2}} = \frac{2x+4}{x}$ .  $(f \circ g)(x)$  is defined whenever both  $g(x)$  and  $f(g(x))$  are defined; that is, whenever  $x \neq 0$  and  $x \neq -2$ . So the domain is  $\{x \mid x \neq 0, -2\}$ .

$(g \circ f)(x) = g\left(\frac{2}{x}\right) = \frac{\frac{2}{x}}{\frac{2}{x} + 2} = \frac{2}{2+2x} = \frac{1}{1+x}$ .  $(g \circ f)(x)$  is defined whenever both  $f(x)$  and  $g(f(x))$  are defined; that is, whenever  $x \neq 0$  and  $x \neq -1$ . So the domain is  $\{x \mid x \neq 0, -1\}$ .

$(f \circ f)(x) = f\left(\frac{2}{x}\right) = \frac{2}{\frac{2}{x}} = x$ .  $(f \circ f)(x)$  is defined whenever both  $f(x)$  and  $f(f(x))$  are defined; that is, whenever  $x \neq 0$ . So the domain is  $\{x \mid x \neq 0\}$ .