Exercises 5.5

1. $r(x) = \dfrac{x-2}{x+3}$. When $x = 0$, we have $r(0) = -\frac{2}{3}$, so the y-intercept is $-\frac{2}{3}$. The numerator is 0 when $x = 2$, so the x-intercept is 2.

3. $t(x) = \dfrac{x^2 - x - 2}{x - 6}$. When $x = 0$, we have $t(0) = \frac{-2}{-6} = 3$, so the y-intercept is 3. The numerator is 0 when $x^2 - x - 2 = (x-2)(x+1) = 0$ or when $x = 2$ or $x = -1$, so the x-intercepts are 2 and -1.

5. $r(x) = \dfrac{x^2 - 9}{x^2}$. Since 0 is not in the domain of $r(x)$, there is no y-intercept. The numerator is 0 when $x^2 - 9 = (x-3)(x+3) = 0$ or when $x = \pm 3$, so the x-intercepts are ± 3.

7. From the graph, the x-intercept is 3, the y-intercept is 3, the vertical asymptote is $x = 2$, and the horizontal asymptote is $y = 2$.

9. $r(x) = \dfrac{5}{x+3}$. There is a vertical asymptote where $x + 3 = 0 \quad \Leftrightarrow \quad x = -3$. We have

$$r(x) = \frac{5}{x+3} = \frac{\frac{5}{x}}{1 + \frac{3}{x}} \to 0 \text{ as } x \to \pm\infty \text{, so the horizontal asymptote is } y = 0.$$

11. $t(x) = \dfrac{x^2}{x^2 - x - 6} = \dfrac{x^2}{(x-3)(x+2)} = \dfrac{1}{1 - \frac{1}{x} - \frac{6}{x^2}} \to 1 \text{ as } x \to \pm\infty$. Hence, the horizontal asymptote is $y = 1$. The vertical asymptotes occur when $(x-3)(x+2) = 0 \quad \Leftrightarrow$ $x = 3$ or $x = -2$, and so the vertical asymptotes are $x = 3$ and $x = -2$.

13. $s(x) = \dfrac{6}{x^2 + 2}$. There is no vertical asymptote since $x^2 + 2$ is never 0. Since

$$s(x) = \frac{6}{x^2+2} = \frac{\frac{6}{x^2}}{1 + \frac{2}{x^2}} \to 0 \text{ as } x \to \pm\infty, \text{ the horizontal asymptote is } y = 0.$$

15. $r(x) = \dfrac{6x - 2}{x^2 + 5x - 6}$. A vertical asymptote occurs when $x^2 + 5x - 6 = (x+6)(x-1) = 0 \quad \Leftrightarrow$ $x = 1$ or $x = -6$. Because the degree of the denominator is greater than the degree of the numerator, the horizontal asymptote is $y = 0$.

17. $y = \dfrac{x^2 + 2}{x - 1}$. A vertical asymptote occurs when $x - 1 = 0 \quad \Leftrightarrow \quad x = 1$. There are no horizontal asymptotes because the degree of the numerator is greater than the degree of the denominator.