Exercises 5.5

2. $s(x) = \dfrac{2x}{3x+5}$. When $x = 0$, we have $s(0) = 0$, so the y-intercept is 0. The numerator is zero when $2x = 0$ or $x = 0$, so the x-intercept is 0.

4. $r(x) = \dfrac{2}{x^2+3x-4}$. When $x = 0$, we have $r(0) = \frac{2}{-4} = -\frac{1}{2}$, so the y-intercept is $-\frac{1}{2}$. The numerator is never zero, so there is no x-intercept.

6. $r(x) = \dfrac{x^3+8}{x^2+4}$. When $x = 0$, we have $r(0) = \frac{8}{4} = 2$, so the y-intercept is 2. The x-intercept occurs when $x^3 + 8 = 0 \quad \Leftrightarrow \quad (x+2)(x^2-2x+4) = 0 \quad \Leftrightarrow \quad x = -2$ or $x = 1 \pm i\sqrt{3}$, which has only one real solution, so the x-intercept is -2.

8. From the graph, the x-intercept is 0, the y-intercept is 0, the horizontal asymptote is $y = 0$, and the vertical asymptotes are $x = -1$ and $x = 2$.

10. $s(x) = \dfrac{3x+3}{x-3} = \dfrac{3+\frac{3}{x}}{1-\frac{3}{x}} \to 3$ as $x \to \pm\infty$. The horizontal asymptote is $y = 3$. There is a vertical asymptote when $x - 3 = 0 \Leftrightarrow x = 3$, so the vertical asymptote is $x = 3$.

12. $r(x) = \dfrac{2x-4}{x^2+2x+1} = \dfrac{\frac{2}{x}-\frac{4}{x^2}}{1+\frac{2}{x}+\frac{1}{x^2}} \to 0$ as $x \to \pm\infty$. Thus, the horizontal asymptote is $y = 0$. Also, $y = \dfrac{2(x-2)}{(x+1)^2}$ so there is a vertical asymptote when $x + 1 = 0 \quad \Leftrightarrow \quad x = -1$, so the vertical asymptote is $x = -1$.

14. $t(x) = \dfrac{(x-1)(x-2)}{(x-3)(x-4)} = \dfrac{x^2-3x+2}{x^2-7x+12} = \dfrac{1-\frac{3}{x}+\frac{2}{x^2}}{1-\frac{7}{x}+\frac{12}{x^2}} \to 1$ as $x \to \pm\infty$, so the horizontal asymptote is $y = 1$. Also, vertical asymptotes occur when $(x-3)(x-4) = 0 \quad \Rightarrow \quad x = 3, 4$, so the two vertical asymptotes are $x = 3$ and $x = 4$.

16. $s(x) = \dfrac{3x^2}{x^2+2x+5} = \dfrac{3}{1+\frac{2}{x}+\frac{5}{x^2}} \to 3$ as $x \to \pm\infty$, so the horizontal asymptote is $y = 3$. Also, vertical asymptotes occur when $x^2 + 2x + 5 = 0 \quad \Rightarrow \quad x = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm 2i$. Since there are no real zeros, there are no vertical asymptotes.

18. $r(x) = \dfrac{x^3+3x^2}{x^2-4} = \dfrac{x^2(x+3)}{(x-2)(x+2)}$. Because the degree of the numerator is greater than the degree of the denominator, the function has no horizontal asymptotes. Two vertical asymptotes occur at $x = 2$ and $x = -2$. By using long division, we see that $r(x) = x + 3 + \dfrac{4x+12}{x^2-4}$ so $y = x + 3$ is a slant asymptote.