

30. Referring to the accompanying figure, use a CAS to estimate the value of k so that the areas of the shaded regions are equal.

Source: This exercise is based on Problem A1 that was posed in the Fifty-Fourth Annual William Lowell Putnam Mathematical Competition.

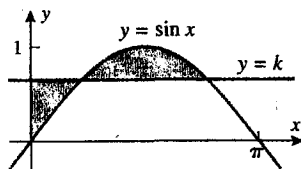


Figure Ex-30

FOCUS ON CONCEPTS

31. Two racers in adjacent lanes move with velocity functions $v_1(t)$ m/s and $v_2(t)$ m/s, respectively. Suppose that the racers are even at time $t = 60$ s. Interpret the value of the integral

$$\int_0^{60} [v_2(t) - v_1(t)] dt$$

in this context.

32. The accompanying figure shows acceleration versus time curves for two cars that move along a straight track, accelerating from rest at the starting line. What does the area A between the curves over the interval $0 \leq t \leq T$ represent? Justify your answer.

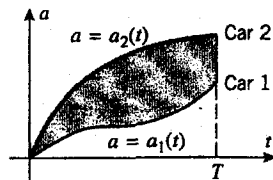


Figure Ex-32

33. Suppose that f and g are integrable on $[a, b]$, but neither $f(x) \geq g(x)$ nor $g(x) \geq f(x)$ holds for all x in $[a, b]$

[i.e., the curves $y = f(x)$ and $y = g(x)$ are intertwined].

- (a) What is the geometric significance of the integral

$$\int_a^b [f(x) - g(x)] dx?$$

- (b) What is the geometric significance of the integral

$$\int_a^b |f(x) - g(x)| dx?$$

34. Let $A(n)$ be the area in the first quadrant enclosed by the curves $y = \sqrt[n]{x}$ and $y = x$.

- (a) By considering how the graph of $y = \sqrt[n]{x}$ changes as n increases, make a conjecture about the limit of $A(n)$ as $n \rightarrow +\infty$.
(b) Confirm your conjecture by calculating the limit.

35. Find the area of the region enclosed between the curve $x^{1/2} + y^{1/2} = a^{1/2}$ and the coordinate axes.

36. Show that the area of the ellipse in the accompanying figure is πab . [Hint: Use a formula from geometry.]

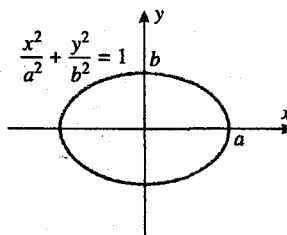


Figure Ex-36

37. A rectangle with edges parallel to the coordinate axes has one vertex at the origin and the diagonally opposite vertex on the curve $y = kx^m$ at the point where $x = b$ ($b > 0$, $k > 0$, and $m \geq 0$). Show that the fraction of the area of the rectangle that lies between the curve and the x -axis depends on m but not on k or b .