

$$38. \frac{1 - \sin x}{1 + \sin x} = \frac{1 - \sin x}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x} = \frac{1 - 2\sin x + \sin^2 x}{1 - \sin^2 x} = \frac{1 - 2\sin x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} - \frac{2\sin x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \sec^2 x - 2\sec x \tan x + \tan^2 x = (\sec x - \tan x)^2$$

$$40. \csc x - \sin x = \frac{1}{\sin x} - \sin x = \frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \cos x \cot x$$

$$42. \sin^4 \theta - \cos^4 \theta = (\sin^2 \theta)^2 - (\cos^2 \theta)^2 = (\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) = \sin^2 \theta - \cos^2 \theta$$

$$44. \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$46. \tan y + \cot y = \frac{\sin y}{\cos y} + \frac{\cos y}{\sin y} = \frac{\sin^2 y + \cos^2 y}{\sin y \cos y} = \frac{1}{\sin y \cos y} = \sec y \csc y$$

$$48. \sin^2 \alpha + \cos^2 \alpha + \tan^2 \alpha = 1 + \tan^2 \alpha = \sec^2 \alpha$$

$$50. \frac{\sin w}{\sin w + \cos w} = \frac{\sin w}{\sin w + \cos w} \cdot \frac{\frac{1}{\cos w}}{\frac{1}{\cos w}} = \frac{\tan w}{1 + \tan w}$$

$$52. \sec t \csc t (\tan t + \cot t) = \frac{1}{\cos t} \cdot \frac{1}{\sin t} \left(\frac{\sin t}{\cos t} + \frac{\cos t}{\sin t} \right) = \frac{1}{\cos^2 t} + \frac{1}{\sin^2 t} = \sec^2 t + \csc^2 t$$

$$54. \frac{1 + \sec^2 x}{1 + \tan^2 x} = \frac{1 + \sec^2 x}{\sec^2 x} = \frac{1}{\sec^2 x} + 1 = \cos^2 x + 1$$

$$56. \frac{\sec x + \csc x}{\tan x + \cot x} = \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} \cdot \frac{\sin x \cos x}{\sin x \cos x} = \frac{\sin x + \cos x}{\sin^2 x + \cos^2 x} = \sin x + \cos x$$

$$58. \frac{\sin A}{1 - \cos A} - \cot A = \frac{\sin A}{1 - \cos A} \cdot \frac{1 + \cos A}{1 + \cos A} - \cot A = \frac{\sin A(1 + \cos A)}{1 - \cos^2 A} - \cot A$$

$$= \frac{\sin A(1 + \cos A)}{\sin^2 A} - \frac{\cos A}{\sin A} = \frac{1}{\sin A} + \frac{\cos A}{\sin A} - \frac{\cos A}{\sin A} = \frac{1}{\sin A} = \csc A$$

$$60. \frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = \frac{1 - \cos x}{\sin x} \cdot \frac{1 - \cos x}{1 - \cos x} + \frac{\sin x}{1 - \cos x} \cdot \frac{\sin x}{\sin x}$$

$$= \frac{1 - 2\cos x + \cos^2 x + \sin^2 x}{\sin x(1 - \cos x)} = \frac{2 - 2\cos x}{\sin x(1 - \cos x)} = \frac{2(1 - \cos x)}{\sin x(1 - \cos x)} = 2 \csc x$$

$$62. \frac{\csc^2 x - \cot^2 x}{\sec^2 x} = \frac{1}{\sec^2 x} = \cos^2 x$$

$$64. \frac{\tan v \sin v}{\tan v + \sin v} = \frac{\tan v \sin v}{\tan v + \sin v} \cdot \frac{\tan v - \sin v}{\tan v - \sin v} = \frac{\tan v \sin v (\tan v - \sin v)}{\tan^2 v - \sin^2 v}$$

$$= \frac{\tan v \sin v (\tan v - \sin v)}{\sin^2 v (\sec^2 v - 1)} = \frac{\tan v \sin v (\tan v - \sin v)}{\tan^2 v \sin^2 v} = \frac{\tan v - \sin v}{\tan v \sin v}$$